Matematická analýza pro informatiky, LS 18/19

Příklady na cvičení 13 (24.5.2019)

Určete globální extrémy funkce $f(x,y,z)$ na množině M.

1. $$f(x,y,z) = x - 2y + 3z,\ M = \{[x,y,z] \in \mathbb{R}^3;\ x^2+y^2 = 1;\ x^2+y^2+z^2 = 2\}.$$

2. $$f(x,y,z) = x - 3y + 2z,\ M = \{[x,y,z] \in \mathbb{R}^3;\ x^2+y^2+z^2 = \frac{59}{4};\ x+y+z = \frac{3}{2}\}.$$

3. $$f(x,y,z) = x^2 + y^2 + z^2,\ M = \{[x,y,z] \in \mathbb{R}^3;\ 2x^2 + y^2 + 3z^2 \leq 6\}.$$

4. $$f(x,y,z) = 4x - y,\ M = \{[x,y,z] \in \mathbb{R}^3;\ y^2 + z^2 = 40;\ x^2 + z^2 = 100\}.$$

5. $$f(x,y,z) = xyz,\ M = \{[x,y,z] \in \mathbb{R}^3;\ x^2 + y^2 + z^2 = 3\}.$$

6. $$f(x,y) = x + y,\ M = \{[x,y] \in \mathbb{R}^2;\ x^3 + y^3 - 2xy = 0;\ x \geq 0;\ y \geq 0\}.$$

Řešení (všichni kandidáti na extrém + hodnoty):

1. $$\begin{aligned} f(\frac{1}{\sqrt{5}}, \frac{-2}{\sqrt{5}}, 1) &= \sqrt{5} + 3\ max, \\ f(\frac{1}{\sqrt{5}}, \frac{-2}{\sqrt{5}}, -1) &= \sqrt{5} - 3, \\ f(\frac{-1}{\sqrt{5}}, \frac{2}{\sqrt{5}}, 1) &= -\sqrt{5} + 3, \\ f(\frac{-1}{\sqrt{5}}, \frac{2}{\sqrt{5}}, -1) &= -\sqrt{5} - 3\ min. \end{aligned}$$

2. $$\begin{aligned} f(-\frac{1}{2}, \frac{7}{2}, -\frac{3}{2}) &= -14\ min, \\ f(\frac{3}{2}, -\frac{5}{2}, \frac{5}{2}) &= 14\ max. \end{aligned}$$

3.

$$\begin{aligned}
f(0,0,0) &= 0\ min,\\
f(\pm\sqrt{3},0,0) &= 3,\\
f(0,\pm\sqrt{6},0) &= 6\ max,\\
f(0,0,\sqrt{2}) &= 2.
\end{aligned}$$

4.

$$\begin{aligned}
f(10,\sqrt{40},0) &= 40-\sqrt{40} \doteq 34,\\
f(10,-\sqrt{40},0) &= 40+\sqrt{40} \doteq 46\ max,\\
f(-10,\sqrt{40},0) &= -40-\sqrt{40} \doteq -46\ min,\\
f(-10,-\sqrt{40},0) &= -40+\sqrt{40} \doteq -34,\\
f(8,2,6)=f(8,2,-6) &= 30,\\
f(-8,-2,6)=f(-8,-2,-6) &= -30.
\end{aligned}$$

5.

$$\begin{aligned}
f(1,1,1)=f(1,-1,-1)=f(-1,1,-1)=f(-1,-1,1) &= 1\ max,\\
f(-1,-1,-1)=f(1,1,-1)=f(1,-1,1)=f(-1,1,1) &= -1\ min.
\end{aligned}$$

6.

$$\begin{aligned}
f(0,0) &= 0\ min\\
f(1,1) &= 2\ max,\\
f(\frac{1}{3}+\frac{\sqrt{3}}{9},\frac{1}{3}-\frac{\sqrt{3}}{9})=f(\frac{1}{3}-\frac{\sqrt{3}}{9},\frac{1}{3}+\frac{\sqrt{3}}{9}) &= \frac{2}{3}.
\end{aligned}$$