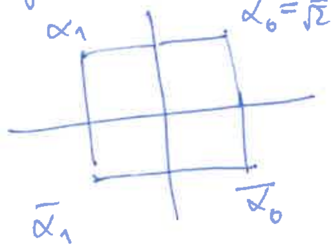
$$\int \frac{dx}{\sin^4 x + \cos^4 x} \quad \left| \begin{array}{ll} y = \operatorname{tg} x & s^2 x = \frac{y^2}{1+y^2} \\ c^2 x = \frac{1}{1+y^2} & x = \operatorname{arctan} y \Rightarrow \end{array} \quad dx = \frac{1}{1+y^2} dy \right|$$

$$= \int \frac{\frac{1}{1+y^2} dy}{\frac{y^4+1}{(1+y^2)^2}} = \int \frac{y^2+1}{y^4+1} dy \Rightarrow \frac{A}{y-\alpha_0} + \frac{\bar{A}}{y-\bar{\alpha}_0} + \frac{B}{y-\alpha_1} + \frac{\bar{B}}{y-\bar{\alpha}_1}$$

$$\alpha_1 = -\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} i = -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} i$$

Kořeny

$\alpha_0 = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} i$



$$\underbrace{-y(\alpha_0 + \bar{\alpha}_0)}_{-\sqrt{2}}$$

sdružit:

$$\frac{A}{y-\alpha_0} + \frac{\bar{A}}{y-\bar{\alpha}_0}$$

Jmenovatel: $y^2 - y\alpha_0 - y\bar{\alpha}_0 + |\alpha_0|^2 = \; = y^2 - y(\sqrt{2}) + 1$

$$\frac{B}{y-\alpha_1} + \frac{\bar{B}}{y-\bar{\alpha}_1}$$

Jmenovatel: $y^2 - y(-\sqrt{2}) + 1$

Čitatelé: $A(y-\bar{\alpha}_0) + \bar{A}(y-\alpha_0)$, tj. lineární, a proto naopak vy. A&B možná složitě početně

obecné lin. fce

$$\frac{Cy+D}{y^2-\sqrt{2}y+1} + \frac{Ey+F}{y^2+\sqrt{2}y+1} = \boxed{\frac{y^2+1}{y^4+1}}$$

$$(Cy+D)(y^2+\sqrt{2}y+1) + (Ey+F)(y^2-\sqrt{2}y+1) = y^2+1$$

Zkusím $C = E = 0$ Pak
$$\left.\begin{array}{l} D+F = 1 \\ D-F = 0 \\ D+F = 1 \end{array}\right\}$$
řešení existuje! $D = F = \frac{1}{2}$

a) $$\int \frac{1}{\underset{\underset{2\cdot 1\cdot\frac{\sqrt{2}}{2}}{\parallel}}{y^2-\sqrt{2}y+1}} dy = \int \frac{1}{(y-\frac{\sqrt{2}}{2})^2+1-\frac{1}{2}} dy \quad \left| z = y - \frac{\sqrt{2}}{2} \right| = \int \frac{1}{z^2+\frac{1}{2}} dz \quad \left| z = \frac{1}{\sqrt{2}} w \right| = \frac{1}{\frac{1}{2}} \int \frac{dw}{w^2+1} \frac{1}{\sqrt{2}}$$

$$= \frac{2}{\sqrt{2}} \operatorname{arctg} w + C = \frac{2}{\sqrt{2}} \operatorname{arctg}(\sqrt{2} z) = \frac{2}{\sqrt{2}} \operatorname{arctg}(\sqrt{2}y - 1) + C$$

b) $$\int \frac{1}{y^2+\sqrt{2}y+1} dy = \dots = \frac{2}{\sqrt{2}} \operatorname{arctg}(\sqrt{2}y+1) + C$$

Celkem: $$\int \frac{1}{\sin^4 x + \cos^4 x} dx = \frac{1}{\sqrt{2}} \left[\operatorname{arctg}(\sqrt{2}y+1) + \operatorname{arctg}(\sqrt{2}y-1)\right] + C$$
$$= \frac{1}{\sqrt{2}} \left[\operatorname{arctg}(\sqrt{2}\tan x + 1) + \operatorname{arctan}(\sqrt{2}\tan x - 1)\right] + C$$

Jiný způsob $y^4+1 = (y^2+ay+b)(y^2+cy+d) =$

$= y^4 + y^3(c+a) + y^2(d+ac+b) + y(ad+bc) + bd$

$\Rightarrow c=-a$, $ad+bc = ad-ab = 0 \Rightarrow a=0$ nebo $\Rightarrow \underline{d=b}$

$d=b \Rightarrow bd = b^2 = 1 \Rightarrow b = \pm 1 = d$

$\pm 1 + ac \pm 1 = 0 \Rightarrow ac = \mp 2 \Rightarrow -a^2 = \mp 2 \Rightarrow a^2 = \pm 2 \Rightarrow$

(a,c ∈ ℝ) $a^2 = 2 \Rightarrow a = \pm\sqrt{2} \Rightarrow c = \mp\sqrt{2}$ a jen $b = +1 = d$

$\Rightarrow (y \pm \sqrt{2}y + 1)(y \mp \sqrt{2}y + 1)$

Pro $a=0$: $c=0 \wedge d+b=0 \Rightarrow d=-b$ $\quad -b^2 = 1$ nemá řeš. v ℝ

Tj. $a=0$ nejde.

Ještě snáze, ale bez zaručení: $y^4+1 = (y^2+ay+1)(y^2+by+1)$?

$= y^4 + (a+b)y^3 + (2-ab)y^2 + (a+b)y + 1$

$\Rightarrow a=-b$ a $2-a^2=0 \Rightarrow a=\pm\sqrt{2} \Rightarrow b=\mp\sqrt{2}$

Tedy lze.

V případě: $((y+D)(y^2+\sqrt{2}y+1)+(Ey+F)(y^2-\sqrt{2}y+1) = y^2+1$

Pokud nechceme tipovat $C=E=0$, řešíme:

$(C+E)y^3 + (\sqrt{2}C + D - \sqrt{2}E + F)y^2 + (C+\sqrt{2}D+E-\sqrt{2}F)y + (D+F) = y^2+1 \Rightarrow$

$C+E=0$, $D+F=1$, $\sqrt{2}C + D + \sqrt{2}C + F = 1$

$2\sqrt{2}C = 1-D-F = 0 \Rightarrow C=0$

$\Rightarrow E=0$; $\sqrt{2}D - \sqrt{2}F = 0 \Rightarrow D=F$, spolu s $D+F=1$

$\Rightarrow D=F=\frac{1}{2}$ ✓