Exercises PDEs II: Set 2 (March 6, 2026)

Sobolev spaces

1. Show that for $\Omega \in C^{0,1}$

$$W^{d,1}(\Omega) \hookrightarrow C_B^0(\Omega),$$

where

$$C_B^0(\Omega) = \{u \in C(\Omega) \mid \sup_{x\in\Omega} |u(x)| =: \|u\|_{C_B^0(\Omega)} < \infty\}.$$

2. Assume that $\Omega \in C^{0,1}$. Simplify the proof of the compact embedding

$$W^{1,p}(\Omega) \hookrightarrow\hookrightarrow L^p(\Omega).$$

3. Prove the Arzelà–Ascoli theorem.
$K \subset C^0(\overline{\Omega})$ is totally bounded, if and only if K is bounded in $C^0(\overline{\Omega})$ and equally uniformly continuous.