DÚ

Nalezněte [illegible] funkci (65)

$u(x,y)$, která splňuje

- $u \in C^1(O)$ O … [illegible] okolí bodu $(1,2)$
- $\frac{\partial u}{\partial x} x^2 + (x^2+y)\frac{\partial u}{\partial y} = 0$ na O

$u(1,y) = u_0(y)$

[illegible]: $1+9$
[illegible]: $6+9$
[illegible]: $6+9$
[illegible]: $3+9$
[illegible]: $5+9$

Řešení:

1 $\frac{dx}{dt} = x^2 \Rightarrow -\frac{1}{x} = C+t$ tj. $x^2 = \frac{1}{(C+t)^2}$

$\frac{dy}{dt} = x^2+y$

$\Rightarrow \frac{dy}{dt} = \frac{1}{(C+t)^2} + y \quad / e^{-t}$

$$\frac{d}{dt}(y e^{-t}) = \frac{e^{-t}}{(C+t)^2}$$

1 $$y(t) = e^t \int_0^t \frac{e^{-s}}{(C+s)^2}\,ds + C_1 e^t$$

$t=0$, … [illegible] (x_0,y_0) $\Rightarrow -\frac{1}{x_0} = C$

$y_0 = C_1 \Rightarrow C_1 = y_0$

1 $C = -\frac{1}{x_0}$

1 $$x(t) = \frac{-1}{t-\frac{1}{x_0}} = \frac{x_0}{1-x_0 t}$$

$$y(t) = e^t \int_0^t \frac{e^{-s}}{(s-\frac{1}{x_0})^2}\,ds + y_0 e^t$$

Hledám hodnotu t, aby platilo $x(t) = 1$

$1 = \frac{x_0}{1-x_0 t}$

$1 - x_0 t = x_0$

$\frac{1}{x_0} - 1 = t$ 1

$$y(t) = e^{-1+\frac{1}{x_0}} \int_0^{\frac{1}{x_0}-1} \frac{e^{-s}}{(s-\frac{1}{x_0})^2}\,ds + y_0 e^{\frac{1}{x_0}-1}$$

$$u(x,y) = u_0\left(e^{\frac{1}{x}-1} \int_0^{\frac{1}{x}-1} \frac{e^{-s}}{(s-\frac{1}{x})^2}\,ds + y e^{\frac{1}{x}-1}\right)$$ 1

Kontrola: $\frac{\partial u}{\partial x} = u_0' \left[-\frac{1}{x^2} e^{\frac{1}{x}-1} \int_0^{\frac{1}{x}-1} \frac{e^{-s}}{(s-\frac{1}{x})^2}\,ds + [illegible] \cdot \left(-\frac{1}{x^2}\right) + e^{\frac{1}{x}-1} \int_0^{\frac{1}{x}-1} \frac{-2e^{-s}}{(s-\frac{1}{x})^3} \frac{1}{x^2}\,ds + y e^{\frac{1}{x}-1} \cdot \left(-\frac{1}{x^2}\right)\right]$

$\frac{\partial u}{\partial y} = u_0' \cdot e^{\frac{1}{x}-1}$

? Přepočítat!

[illegible]

(2) (95) Rozwiąż równanie

$x\partial_x u + y\partial_y u = u^2$

spełniające $u(1,y) = y^2$ na dziedzinie [...]

Rozw.

Rozważamy liniowe PDE

$x\frac{\partial w}{\partial x} + y\frac{\partial w}{\partial y} + z^2\frac{\partial w}{\partial z} = 0$ 1

Charakterystyki

1 $\frac{dx}{dt} = x$, $\frac{dy}{dt} = y$, $\frac{dz}{dt} = z^2$ $\Rightarrow$ 1 $\frac{d}{dt}(\ln x - \ln y) = 0$

1 $\frac{d}{dt}(\ln x + \frac{1}{z}) = 0$

1 $u(x,y,z) = U(\ln x - \ln y;\ \ln x + \frac{1}{z})$

a spełnia:

$U(-\ln y, \frac{1}{y^2}) = 0$

1 $e^{-2t_1} - \frac{1}{(t_2)} = 0$

1 $e^{-2(\ln x - \ln y)} - \frac{1}{\ln x + \frac{1}{u}} = 0$

1 $\frac{y^2}{x^2} = \frac{1}{\ln x + \frac{1}{u}}$

$\ln x + \frac{1}{u} = \frac{x^2}{y^2}$

1 $\frac{1}{u} = \frac{x^2}{y^2} - \ln x$

1 $u = \frac{y^2}{x^2 - y^2\ln x}$

$x\frac{\partial u}{\partial x} = \frac{-y^2}{(x^2 - y^2\ln x)^2}\left(2x - \frac{y^2}{x}\right)\cdot x$

$y\frac{\partial u}{\partial y} = \frac{2y\cdot y}{x^2 - y^2\ln x} + \frac{y^2}{(x^2 - y^2\ln x)^2}\cdot 2y^2\ln x$

$\frac{-2x^2y^2 + y^4 + 2y^2(x^2 - y^2\ln x) + 2y^4\ln x}{(x^2 - y^2\ln x)^2} = u^2$ ✓