Orlicz-Sobolevskii spaces on the boundary of a domain. The trace operator, extension and heat kernel

Agnieszka Kałamajska
Institute of Mathematics, Warsaw University
email: kalamajs@mimuw.edu.pl

joint works with **Miroslav Krbec** *from Academy of Sciences of the Czech Republic*

Abstract

Let Ω be a bounded subset of $\mathbf{R}^n$ with Lipschitz boundary, $\Psi, \Phi : [0, \infty) \to [0, \infty)$ be N-functions (in particular convex). By $W^{1,\Phi}(\Omega)$ we denote the space of functions defined on Ω, who together with their weak derivatives are in Orlicz space $L^{\Phi}(\Omega)$. We are also dealing with Orlicz variant of Besov-Slobodecki space $Y^{\Psi,\Phi}(\partial\Omega)$, being the linear hull of all $u \in L^{\Psi}(\partial\Omega)$, for which the seminorm

$$I^{\Phi}(u, \partial\Omega) := \int_{\partial\Omega} \int_{\partial\Omega} \Phi\left(\frac{|u(x) - u(y)|}{|x - y|}\right) \frac{1}{|x - y|^{n-2}} \, d\sigma(x) d\sigma(y)$$

is finite, where σ is the $n-1$-dimensional Lebegue measure defined on $\partial\Omega$. It is equipped with the related Luxemburg-type norm. When $\Phi(\lambda) = \Psi(\lambda) = \lambda^p$, $p > 1$, they generalize the known Sobolev-Slobodetskii space $W_p^{1-1/p}(\partial\Omega)$.

We define trace operator $Tr : W^{1,\Phi}(\Omega) \to Y^{\Psi,\Phi}(\partial\Omega)$, under suitable conditions upon N-functions Ψ, Φ and discuss its surjectivity. Our approach extends the trace theorems for $W^{1,\Phi}(\Omega)$, established in Orlicz setting by Lacroix and Palmieri in late 70-ties, where the complementary Young function to Φ had to fulfill the Δ_2-condition. In our case this assumption is relaxed. Arguments are based on suitable variants of Hardy inequalities and on properties of heat kernel.

Presentation is based on recently obtained joint works with Miroslav Krbec.