Let A and A' be two circular annuli and let ρ be a radial metric defined in the annulus A'. Consider the class $\mathcal{H}_\rho$ of $\rho-$harmonic mappings between A and A'. It is proved recently by Iwaniec, Kovalev and Onninen that, if $\rho = 1$ (i.e. if ρ is Euclidean metric) then $\mathcal{H}_\rho$ is not empty if and only if there holds the Nitsche condition (and thus is proved the J. C. C. Nitsche conjecture). In this paper we formulate a condition (which we call $\rho-$Nitsche conjecture) with corresponds to $\mathcal{H}_\rho$ and define $\rho-$Nitsche harmonic maps. We determine the extremal mappings with smallest mean distortion for mappings of annuli w.r. to the metric ρ. As a corollary, we find that $\rho-$Nitsche harmonic maps are Dirichlet minimizers among all homeomorphisms $h : A \to A'$. However, outside the ρ-Nitsche condition of the modulus of the annuli, within the class of homeomorphisms, no such energy minimizers exist. This extends some recent results of Astala, Iwaniec and Martin (ARMA, 2010) where it is considered the case $\rho = 1$ and $\rho = 1/|z|$.