Dynamic Bayesian Estimation in Diffusion Networks

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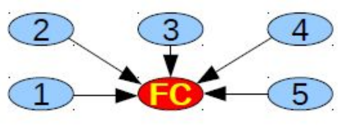
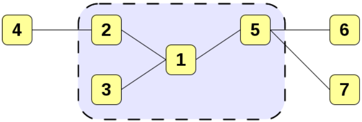
Introduction

- Our aim is to solve the task of collaborative estimation of unknown environmental parameter from noisy measurements.
- We focus on fully decentralized collaborative estimation in networks allowing the nodes to communicate only with their adjacent neighbours:



(a) Centralized case



(b) Decentralized case

Existing algorithms for estimation in distributed networks

Bayesian methods: treating general tasks with distributed character from the decision making perspective, ranging from [Tsitsiklis and Athans(1982)] to [Aysal and Barner(2008)].

Non-Bayesian methods for fully decentralized collaborative estimation (mostly single problem oriented, e.g., on):

- least-squares estimation [Xiao, Boyd, and Lall(2006)],
- recursive least-squares (RLS, [Cattivelli, Lopes, and Sayed(2008)]),
- least mean squares (LMS, [Cattivelli and Sayed(2010)]),
- Kalman filters ([Cattivelli, Lopes, and Sayed(2008)]) etc.

The dynamic Bayesian estimation in diffusion networks

Nodes collectively estimate the common parameter of interest using the same model structure. Furthermore, they satisfy the following constraint: *the nodes are able to communicate one-to-one only within their closed neighbourhood*.
Tools such a:

- Bayesian decision theory,
- Kullback Leibler divergence,
- minimum cross entropy principle

yield a theoretically consistent incremental update, which is guaranteed by the principle of weighted likelihoods [Wang(2004), Wang(2006)].

Basic steps

Basic steps of the proposed dynamic Bayesian estimation are:

Incremental update – also known as the data update. The nodes propagate data within their closed neighbourhood and incorporate them into their local statistical knowledge;

Spatial update – the nodes propagate point parameter estimates (i.e. mean values) or posterior pdfs within their closed neighbourhood and correct their local estimates.

Basic formulas

Bayesian recursive estimation:

$$g(\boldsymbol{\Theta}|\boldsymbol{d}(t)) \propto f(y_t|u_t, \boldsymbol{d}(t-1), \boldsymbol{\Theta})g(\boldsymbol{\Theta}|\boldsymbol{d}(t-1)).$$

The incremental update:

$$\begin{aligned} g_k(\boldsymbol{\Theta}|\overline{\boldsymbol{d}}(t)) &\propto g_k(\boldsymbol{\Theta}|\overline{\boldsymbol{d}}(t-1)) \\ &\times \prod_{l \in \mathcal{N}_k} f_l(y_{l,t}|u_{l,t}, \boldsymbol{d}_l(t-1), \boldsymbol{\Theta})^{c_{l,k}}, \end{aligned}$$

$c_{l,k}$ are given weights representing the weight of lth node with respect to the kth one, $\sum_{l \in \mathcal{N}_k} c_{l,k} = 1$.

The spatial update:

$$g_k(\boldsymbol{\Theta}|\overline{\boldsymbol{d}}(t)) = \sum_{l \in \mathcal{N}_k} a_{l,k} g_l(\boldsymbol{\Theta}|\overline{\boldsymbol{d}}(t)), \quad \sum_{l \in \mathcal{N}_k} a_{l,k} = 1,$$

$0 \leq a_{l,k} \leq 1$ is the weight of lth node's estimate from kth node's viewpoint.

Further details and example regarding Gaussian linear regressive model can be found on the yellow poster.

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