Linear SEE's with additive noise of Volterra type

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ROBUST 2016
September 13, 2016

The problem

The stochastic Cauchy problem

$$
\begin{aligned}
dX_t &= AX_t\,dt + \Phi\,dB_t \quad t \in [0,T] \\
X_0 &= x
\end{aligned}
$$

U, V	...	real, separable Hilbert spaces
$A : \mathrm{Dom}(A) \subset V \to V$	...	generates a C_0-semigroup of operators on V
$B = (B_t, t \in [0,T])$	...	infinite-dimensional noise in U
$\Phi \in \mathcal{L}(U,V)$	...	diffusion term
$x \in V$	...	initial condition

Typical Examples

Stochastic heat equation:

$$\begin{aligned} \partial_t u &= \Delta u + \eta \quad \text{on } [0,T]\times\mathcal{O} \\ u(0,\cdot) &= f \quad \text{on } \mathcal{O} \\ u|_{[0,T]\times\partial\mathcal{O}} &= 0 \end{aligned}$$

Stochastic wave equation:

$$\begin{aligned} \partial_{tt}^2 u &= \Delta u + \eta \quad \text{on } [0,T]\times\mathcal{O} \\ u(0,\cdot) &= f_1 \quad \text{on } \mathcal{O} \\ \partial_t u(0,\cdot) &= f_2 \quad \text{on } \mathcal{O} \\ u|_{[0,T]\times\partial O} &= 0 \end{aligned}$$

$\mathcal{O}\subset\mathbb{R}^d$	...	bounded domain with smooth boundary
f, f_1, f_2	...	given functions
η	...	space-time noise process

Mild solution

Stochastic evolution equation (SEE):

$$\begin{aligned} dX_t &= AX_t\,dt + \Phi\,dB_t \quad t \in [0,T] \\ X_0 &= x \end{aligned}$$

Mild solution:

$$X_t = S(t)x + \int_0^t S(t-r)\Phi\,dB_t \quad t \in [0,T]$$

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- give meaning to the convolution integral
- prove that $X = (X_t, t \in [0,T])$ has measurable sample paths
- prove that X has continuous sample paths

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But how to define the integral when B is neither a semimartingale, nor a Gaussian process?

Volterra processes

K ... so-called regular Volterra kernel (see poster for the definition)

Definition

A stochastic process $b = (b_t, t \in [0,T])$ is called a Volterra process if

(i) b is centered with $b_0 = 0$

(ii) b is ε-Hölder continuous for every $\varepsilon \in (0,\delta)$ for some $\delta > 0$

(iii) the covariance of b is

$$R(s,t) = \int_0^{s\wedge t} K(s,r)K(t,r)dr, \quad s,t \in [0,T].$$

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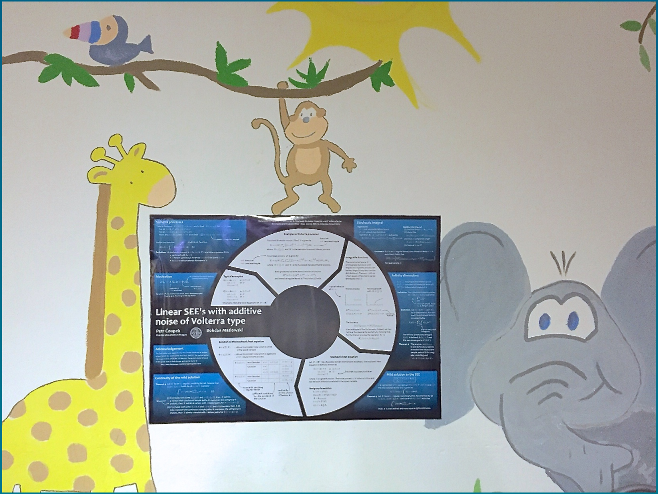
Examples:

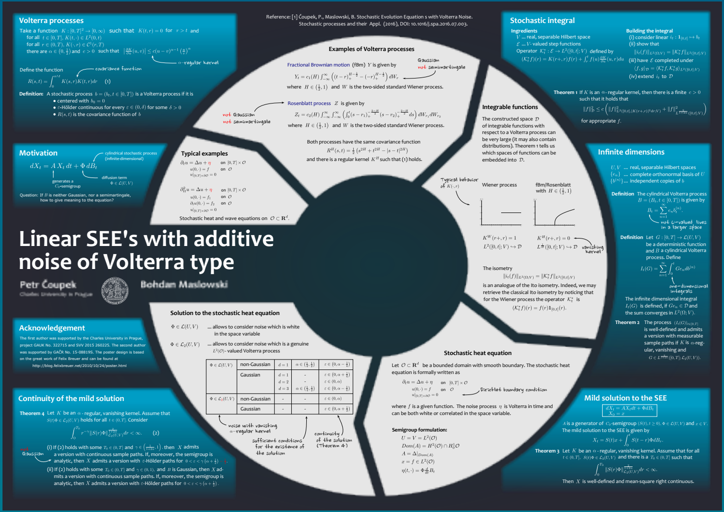
- fractional Brownian motion of $H > \frac{1}{2}$
- Rosenblatt process

The poster

Where to find the poster



How to read the poster



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