Estimation of parameters in a planar segment process

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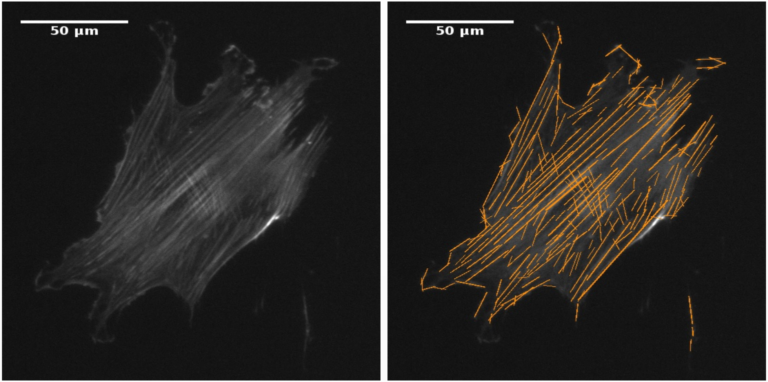
Studied data

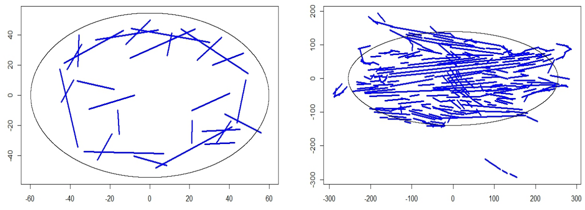
- Human stem cells
- Actin stress fibres
- Filament sensor algorithm (Eltzner et al. 2016)

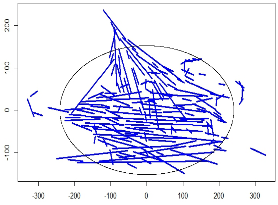


Stem cells data

- Cell approximated by ellipse
- Three categories of cells - they have been placed on gel with different stiffness
- Stiffness of gel - precursor for cell differentiation
  - 1 kPa - neuron cells,
  - 10 kPa - muscle cells,
  - 30 kPa - bone cells
- Stiffness measured - Young modulus

Stem cells data categories

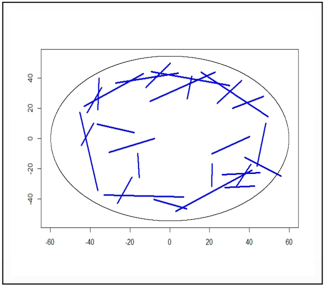


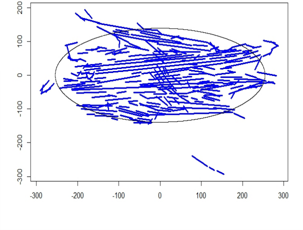


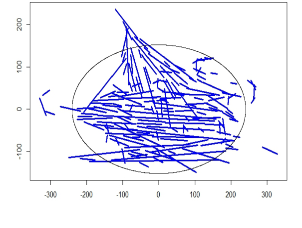
Problems

- 1 kPa more chaotic structure - easier for modeling
  - firstly - only this category
  - restriction - only cells well approximated by ellipse
- 10 kPa and 30 kPa cells
  - very complex structure
  - usually not well approximated by ellipse
  - saving these data for later

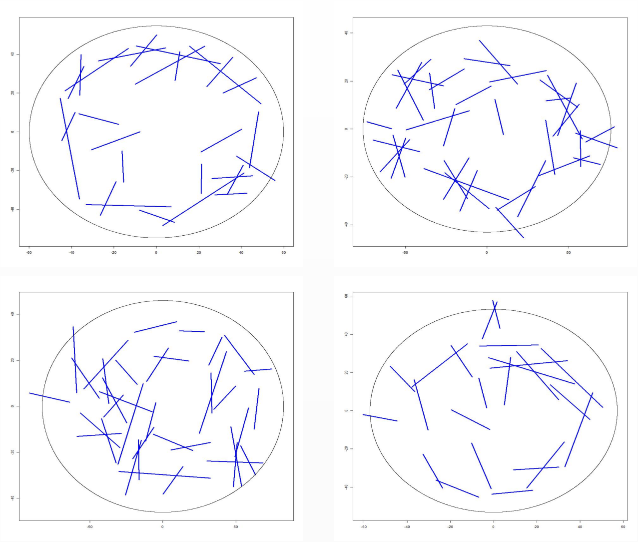
Stem cells data categories







1 kPa cells examples



Model for data

Model should reflect following features

- Expects only fibers in ellipse approximation
- Fibers are driven by Poisson process
- Fibers length is driven by beta distribution
- Segments further from centre appear with higher probability

Model for data

We consider probability density with respect to the Poisson process with unit intensity

$$p(\mathbf{x}) = c 1_{\mathbf{x} \subset B} \exp\left(b \sum_{u \in \mathbf{x}} d(u) \right) (z|B|)^{n(\mathbf{x})} \prod_{u=(r,x) \in \mathbf{x}} g\left(\frac{r}{e_1} \right)$$

- where B is ellipse approximation of cell, e_1 size of longer axis, $\mathbf{x}$ is a realization of a process, $d(u)$ is the largest distance of fiber u from the ellipse center, $n(\mathbf{x})$ is number of fibers in $\mathbf{x}$.
- $g(r) = \frac{r^{\alpha-1}(1-r)^{\beta-1}}{B(\alpha,\beta)}$
- We have parameters b, z, α and β to estimate.
- c is constant.

- Takacz-Fiksel method based on Georgii-Nguyen-Zessin formula:
- $\mathbb{E}\sum_{u\in\mathbf{x}} q(u,\mathbf{x}\setminus u) = \int \lambda^*(\mathbf{x},u)q(u,\mathbf{x})du$, where λ^* is Papangelou intensity in form

$$\lambda^*(\mathbf{x},u) = \lambda^*(u) = \lambda^*((r,x)) = z\mathbf{1}_{u\subset B}\exp\left(bd(u)\right)g\left(\frac{r}{e_1}\right)$$

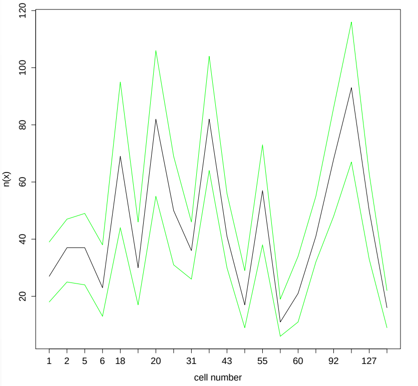
- We minimize $\left(\sum_{u\in\mathbf{x}} q(u,\mathbf{x}\setminus u) - \int \lambda^*(u)q(u,\mathbf{x})du\right)^2$ for selected functions $d(u)$, $\log\left(\frac{l(u)}{e_1}\right)$, $\log\left(1-\frac{l(u)}{e_1}\right)$, 1.

Estimation results

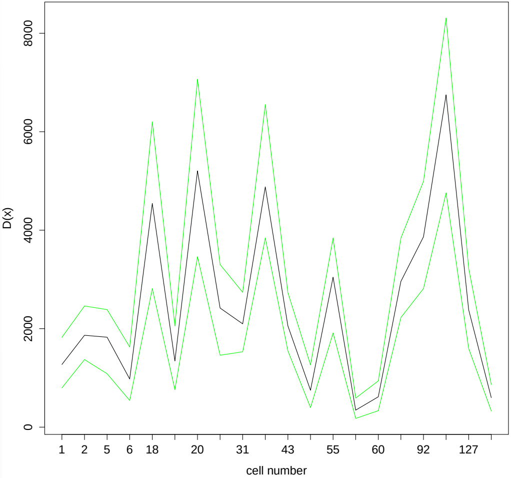
No	e_1	e_2	$z.10^3$	$b.10^2$	α	β	$I.10^2$
001	59	54	0.46	4.99	1.76	4.27	0.83
002	76	43	1.67	2.88	1.58	4.35	1.13
005	83	46	4.49	0.45	1.59	4.50	0.97
006	57	53	1.98	1.70	2.00	4.47	0.76
018	97	69	1.77	1.65	1.34	4.95	1.03
019	62	46	1.12	3.97	1.87	4.05	1.05
020	94	78	4.07	0.41	1.41	4.67	1.12
030	69	57	3.54	1.13	1.75	4.38	1.27
031	88	50	1.18	2.40	1.44	4.46	0.81
034	92	83	9.44	-1.05	1.43	4.71	1.07
043	67	64	2.43	1.38	1.81	4.52	0.95
049	63	56	2.54	-0.02	1.86	4.34	0.48
055	72	66	2.25	1.85	1.91	5.22	1.20
059	49	45	19.03	-5.41	2.74	4.42	0.49
060	46	34	19.10	-2.57	2.84	4.92	1.31
064	109	89	1.48	0.28	1.29	5.75	0.42
092	86	80	7.89	-1.0	1.69	5.91	0.98
093	108	76	1.98	1.42	1.25	4.63	1.13
127	70	65	7.63	-0.6	1.65	4.28	1.13
131	48	34	0.14	10.26	5.73	12.3	0.90

Verification results

- Simulation from the estimated model
- 19 simulations for every cell from which we create confidence interval for selected statistics $D(\mathbf{x}) = \sum_{u \in \mathbf{x}} d(u)$ and $n(\mathbf{x})$ by taking minimum and maximum values of statistics from simulations.
- We can see that values of statistics lie within the interval



Verification results



Bright future

- Create more complex model to include also cells from 10 kPa and 30 kPa category.
- Include to modelling cells, which are not well approximated by ellipsis.
- Differentiate cell categories according to estimated model parameters.

References

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