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Maximum Volatility Portfolio

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The investor is allowed to invest to assets $S_1, \ldots, S_{n+1}$ with price evolution for $i \neq j$

$$dS_{ij}(t) = S_{ij}(t)\sigma_{ij}^T(t)dW^j(t) \;\; and \;\; S_{ij}(0) = 1,$$

where $\sigma_{ij} = \left(\sigma_{ij1}, \ldots, \sigma_{ijn}\right)^T$ satisfies for every $T > 0$ the integrability condition $\int_0^T \sigma_{ij}^T(t)\sigma_{ij}(t)dt < \infty$ *a.s.* and $W^j = \left(W^{j1}, \ldots, W^{jn}\right)^T$ is an n-dimensional Brownian motion. To simplify formulas we set $\sigma_{iik} = 0$ for every i and k.

Definition of the model II

We define the index of the market as

$$I(t) = S_1(t) + \cdots + S_{n+1}(t),$$

and the portfolio as

$$X(t) = \Delta_1(t)S_1(t) + \cdots + \Delta_{n+1}(t)S_{n+1}(t),$$

where Δ_i represents number of units of asset S_i held by the investor at time t.

Evolution of portfolio

It can be shown, that the price of the portfolio X with respect to the index I follows stochastic differential equation

$$dS_{XI}(t) = \sum_{i=1}^{n+1} \sum_{j=1}^{n+1} S_{jI}(t) S_{iI}(t) \Delta_i(t) \sigma_{ij}^T(t) dW^I(t).$$

Independence on the choice of parameters

Theorem. Independence on choice of σ's

Suppose that volatilities of price processes S_{ij} are known for fixed j. That is, we know $\sum_{k=1}^{n} \sigma_{ijk}^2$ for every i. Then independently of choice of specific estimates of σ_{ijk} the models are equivalent.

The buyer of the passport option pays a premium upfront and is allowed to actively trade with no shorting allowed. At the strike time T he keeps the trading profits while he is forgiven any loss. Mathematically, the holder of the option wants to maximize

$$\mathbb{E}^I\left[(S_{XI}(T)-1)^+\right],$$

where $\mathbb{E}^I$ is expected value with respect to martingale measure of the asset I and

$$\Delta_i(t) \geq 0, \quad t \in [0,T],\ i \in \{1,\dots,n+1\}.$$

Theorem. Optimal strategy for the option for two assets

The strategy that maximizes $\mathbb{E}^I[(S_{XI}(T) - 1)^+]$ is being fully invested in the cheaper asset.

Properties of the optimal strategy

- The optimal strategy is equivalent to maximizing $\mathbb{E}^I[\langle S_{XI}\rangle_T]$;
- Is also optimal for every convex payoff function;
- Slightly modified optimal strategy maximizes probability of outperforming index I by $\alpha > 0$ at time T.

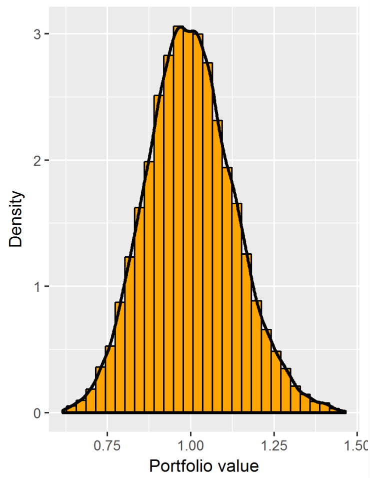
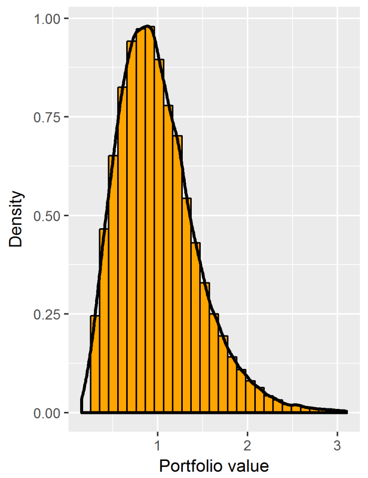


Figure: Histogram for 50 000 simulations with parameters $T = 1$



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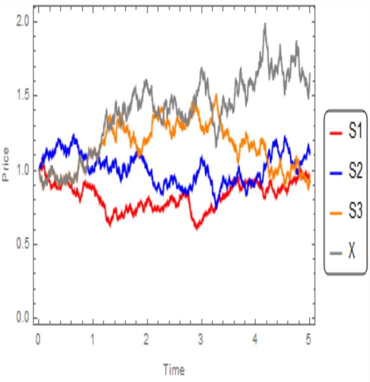

Figure: Evolutions with respect to index.

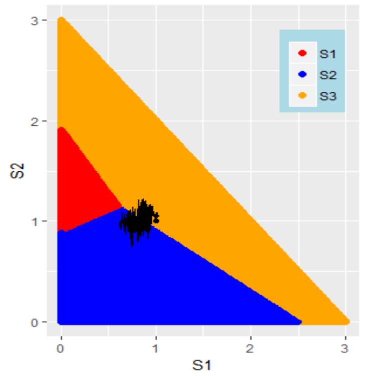

Figure: Optimality regions of assets with evolution of (S_{1l}, S_{2l}).

Used parameters: $T = 5$, $volatility(S_{12}) = 0.3$, $volatility(S_{13}) = 0.35$ and $volatility(S_{32}) = 0.4$.

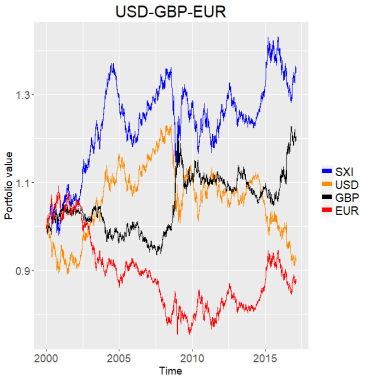

Figure: Evolution of the portfolio and USD, GBP, and EUR.

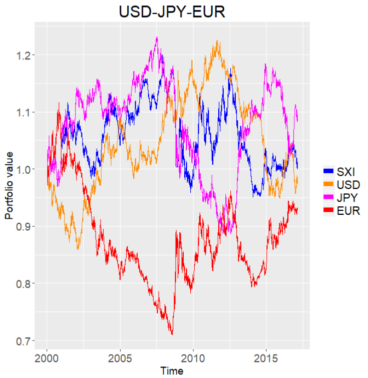

Figure: Evolution of the portfolio and USD, JPY, and EUR.

Application of the optimal strategy on currencies.

Thank you for your attention!