6 Primitive polynomials, Gauss lemma

6.1. Find irreducible decompositions in the domains $\mathbb{R}[x]$, $\mathbb{C}[x]$, $\mathbb{Z}[x]$ of polynomials

(a) $6x - 6$,

(b) $2x^2 + 2$.

6.2. Calculate $\gcd(f, g)$ for given polynomials f and g:

(a) $f = 6x^3 - 6$, $g = 8x^2 - 8$ in $\mathbb{Z}[x]$

(b) $f = 6x^2 + 3x - 3$, $g = 6x^2 + 6x$ in $\mathbb{Q}[x]$

(c) $f = 6x^2y$, $g = 15xy^2 + x^3y$ in $\mathbb{Z}[x, y]$.

6.3. Find all rational roots of the given polynomials in $\mathbb{Z}[x]$:

(a) $3x^5 - 2x^2 + x + 1$,

(b) $x^3 - 7x^2 + 11x + 3$,

(c) $2x^3 - x^2 + 3$.

6.4. Prove that the following polynomials are irreducible:

(a) $x^3 + x^2 + x + 3$ in $\mathbb{Z}[x]$,

(b) $4x^3 - 15x^2 + 60x + 180$ in $\mathbb{Z}[x]$,

(c) $\frac{10}{17}x^8 + 5x^6 + \frac{9}{2}x^5 - 12x^4 + \frac{4}{3}x - 6$ in $\mathbb{Q}[x]$.

6.5. Decompose the polynomial $2x^2 + 2x - 1$ into a product of irreducible polynomials in $\mathbb{Z}[\sqrt{3}][x]$.