9 Finite Fields and Some Applications

9.1. Construct a splitting field of the polynomials

(a) $x^3 + 1$ over the field $\mathbb{Z}_2$,

(b) $x^2 + 1$ over the field $\mathbb{Z}_7$

and decompose all the polynomials into linear factors.

9.2. Construct finite fields consisting of (a) 25, (b) 8, (c) 125 elements.

9.3. Prove that $\mathbb{Z}_3[\alpha]/(\alpha^4 + \alpha^3 + \alpha + 2)$ is not a field.

9.4. Design a secret sharing protocol for 5 participants such that at least 3 of them are needed to reveal the secret where the secret is an element of the field $\mathbb{F}_7$.

9.5. At a competition, athletes from five continents participated. From each continent there is exactly one representative in each of the five disciplines. Draw a 5×5 arrangement of athletes such that in every row and every column there is exactly one athlete from each continent and exactly one athlete from each discipline, and moreover, all athletes on the main diagonal are Europeans.