

Homework 3

Deadline: Tuesday, October 29 at 14:00.

Please submit your solutions either on paper at the beginning of the practicals or as a pdf-file in <https://owl.mff.cuni.cz/> Everything that is not immediately obvious needs to be proved or quoted from lecture notes.

Note that you should solve only one of the following questions below.

1. Let X be a set, $|X| > 1$, denote $P(X) = \{Y : Y \subseteq X\}$, and $A \div B = (A \cup B) \setminus (A \cap B)$ and $-A = A$ for each $A, B \subseteq X$.
 - (a) Prove that $(P(X), \div, -, \cap, \emptyset)$ is a commutative ring.
 - (b) What is identity of the operation $\cap$?
 - (c) Is $(P(X), \div, -, \cap, \emptyset)$ domain?
2. Let $e \in \mathcal{R}$ be an idempotent element of a ring $\mathcal{R}$. Determine whether the set $e\mathcal{R}e = \{eae : a \in \mathcal{R}\}$ with the operations addition and multiplication inherited from $\mathcal{R}$ is a ring, and whether it is a subring of $\mathcal{R}$

Note: An element e is idempotent if $e^2 = e$.