Metametaquestions in Constraint Tractability

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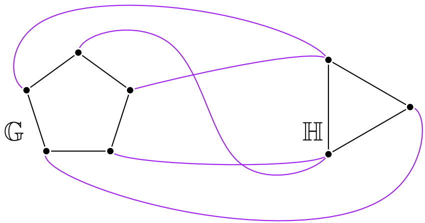
Funded by the European Union (ERC, POCOCOP, 101071674). Views and opinions expressed are however those of the author(s) only and do not necessarily reflect those of the European Union or the European Research Council Executive Agency. Neither the European Union nor the granting authority can be held responsible for them.

Constraint Satisfaction Problems: definition



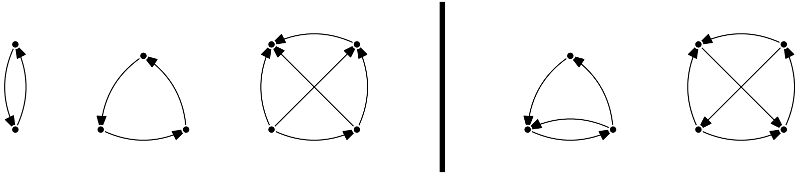
Let $\mathbb{H}$ be a relational structure (e.g. graph, digraph, hypergraph)

Homomorphism from $\mathbb{G}$ to $\mathbb{H}$ is a mapping $h\colon G \to H$ which preserves the relations: $\forall \bar{t} \in R^{\mathbb{G}}\ h(\bar{t}) \in R^{\mathbb{H}}$, denoted $h\colon \mathbb{G} \to \mathbb{H}$

For fixed $\mathbb{H}$, the *Constraint Satisfaction Problem* CSP($\mathbb{H}$) is a decision problem asking for a structure $\mathbb{G}$ if there is a homomorphism $h\colon \mathbb{G} \to \mathbb{H}$

Constraint Satisfaction Problems: complexity

Bang-Jensen, Hell, MacGillivray (1988): If semicomplete digraph $\mathbb{H}$ has ≤ 1 directed cycle, then CSP($\mathbb{H}$) is tractable; otherwise it is NP-complete



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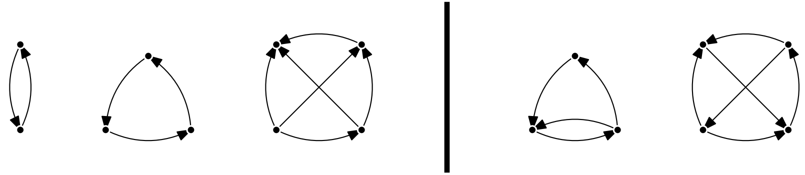
$$\forall u, v \left[u \neq v \Leftrightarrow (u \to v \vee v \to u) \right]$$



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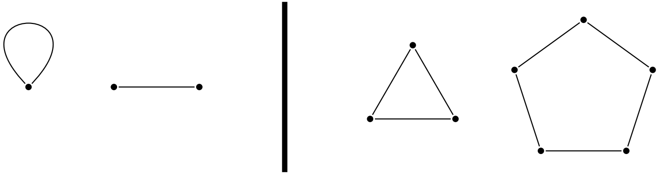

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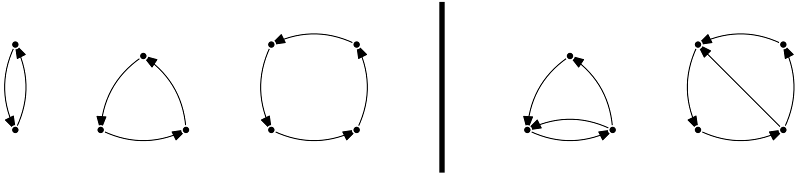

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Polymorphism (m-ary) of $\mathbb{H}$ is a mapping $f\colon H^m \to H$ preserving the relations component-wise: $\forall \bar{t}_1, \ldots, \bar{t}_m \in R^{\mathbb{H}}\ h(\bar{t}_1, \ldots, \bar{t}_m) \in R^{\mathbb{H}}$, denoted $f\colon \mathbb{H}^m \to \mathbb{H}$

Bulatov (2017), Zhuk (2017): If digraph $\mathbb{H}$ has $f\colon \mathbb{H}^m \to \mathbb{H}$ such that

$$\forall x_1, \ldots, x_m \in H \quad f(x_1, \ldots, x_m) = f(x_2, \ldots, x_m, x_1),$$

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"structural"

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then CSP($\mathbb{H}$) is tractable; otherwise CSP($\mathbb{H}$) is NP-complete "algebraic"

Beyond: matrix partitions

Let M be a symmetric $(n \times n)$-matrix with values in $\{0, 1, *\}$

For fixed M, the *M-partition problem* asks for graph $\mathbb{G} = (V, E)$ if exists a mapping $p\colon V \to [n]$ such that for all distinct $u, v \in V$:

- if $uv \notin E$, then $M_{p(u)p(v)} \in \{0, *\}$
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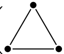
same as CSP()

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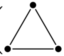
same as CSP()

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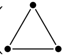
same as CSP()

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Hell: are M-partitions also in P $\cup$ NP-complete? If so, what is the classification?

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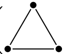
same as CSP()

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Let Π be some property on graphs

Sandwich problem over Π takes as input a pair of graphs $(V, E_1), (V, E_2)$ such that $E_1 \subseteq E_2$, and asks to find $E_1 \subseteq E \subseteq E_2$ such that (V, E) satisfies Π

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Observation:

$$\begin{aligned} \Pi &\leq_p \text{Sandwich } \Pi \\ (V, E) &\mapsto [(V, E), (V, E)] \end{aligned}$$

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Motivation:

- a "real world" problem
- unclear complexity: there exist NP-complete, coNP-complete & coNP-intermediate sandwich problems

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Bodirsky, Guzmán-Pro (2026): Sandwich Π is an infinite-domain CSP for many well-known graph-theoretic properties Π

Sandwich Matrix Partitions

Let M be a symmetric $(n \times n)$-matrix with entries from $\{0, 1, *\}$
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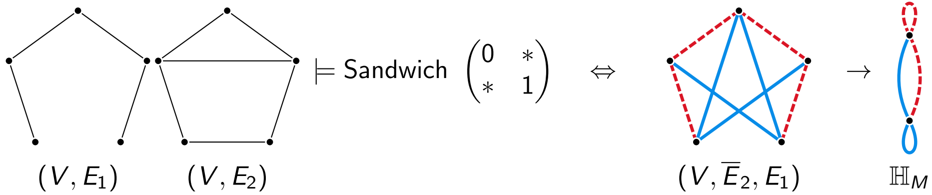

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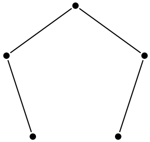

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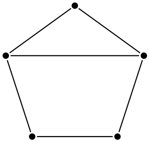

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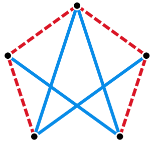

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$\mathbb{H}_M$ – reflexive complete 2-edge-colored graph

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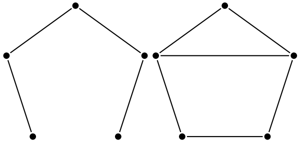

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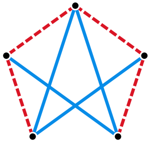

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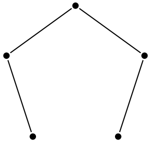

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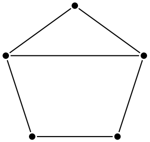

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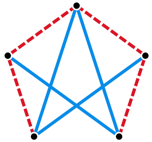

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Are we happy?

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Every CSP is poly-time equivalent to CSP($\mathbb{H}$) where

$\mathbb{H} = (H, E)$ – digraph

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graph	$\forall x, y \; Exy \to Eyx$	has loop or is bipartite	P
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For reflexive complete 2-edge-colored $\mathbb{G}, \mathbb{H}$, a *homogeneous concatenation* $\mathbb{G} \triangleleft \mathbb{H}$ is obtained from $\mathbb{G} \sqcup \mathbb{H}$ by adding edges as follows:
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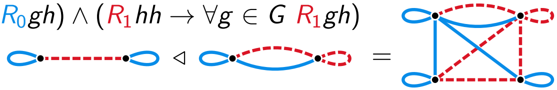

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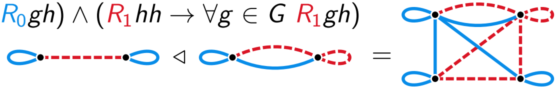

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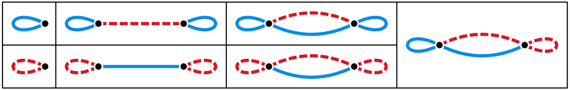

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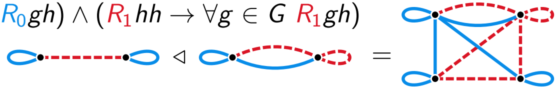

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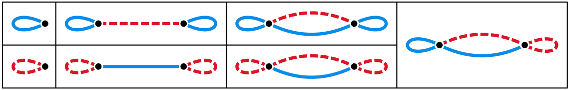

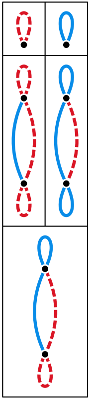

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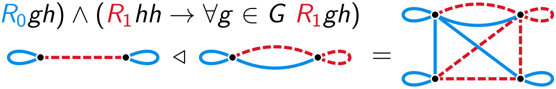

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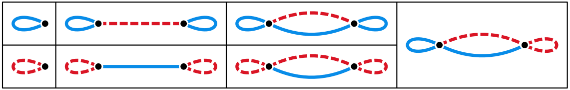

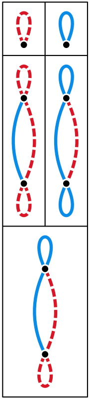

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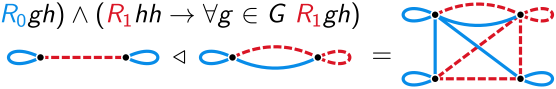

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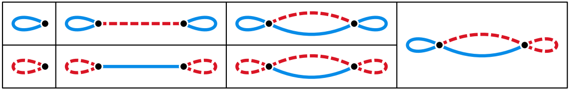

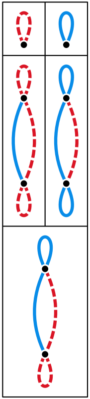

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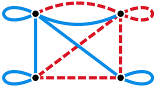

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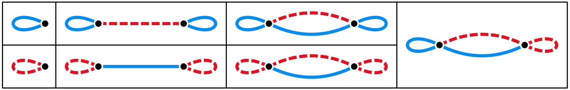

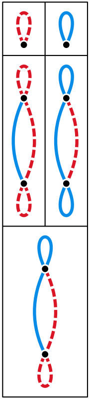

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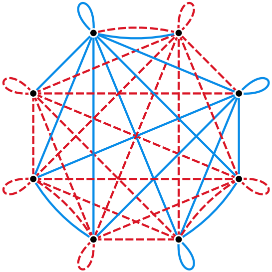
Moreover: every such tractable CSP has bounded width (as well as all other classes with "structural" classification)

Example

What is the complexity of CSP($\mathbb{H}$)?



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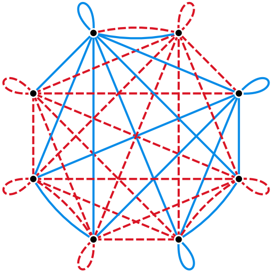

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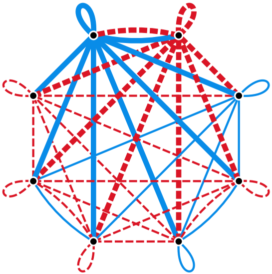

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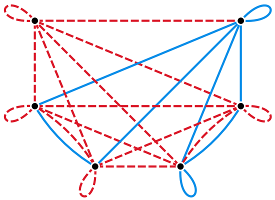

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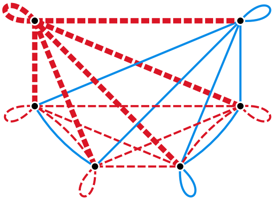

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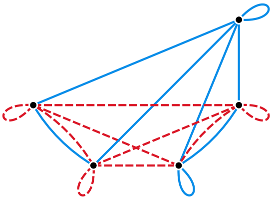

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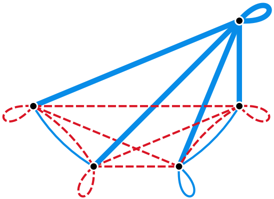

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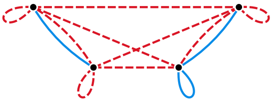

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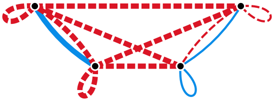

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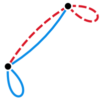

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