Containment for Guarded Monotone Strict NP

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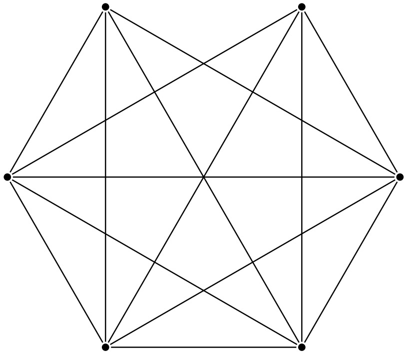
What is GMSNP?

A classic example

No Monochromatic Triangle

Given: a graph (V, E)
Task: to partition E in two classes E_1, E_2 such that neither (V, E_1) nor (V, E_2) contains a triangle



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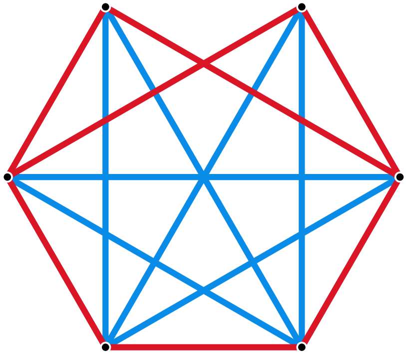
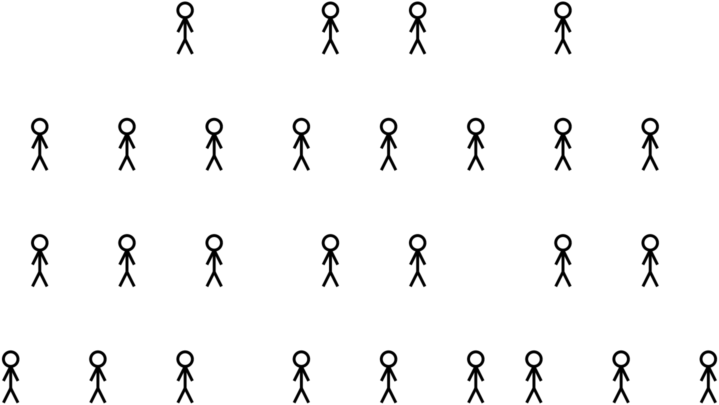


(V, E_1, E_2)

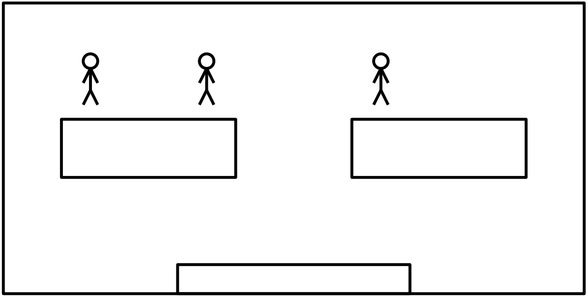
A “real life” example



ICALP participants

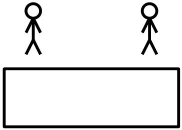
A “real life” example

Lecture hall

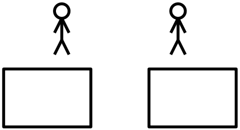


Two desks, three people

A “real life” example



$\Longrightarrow$ always together



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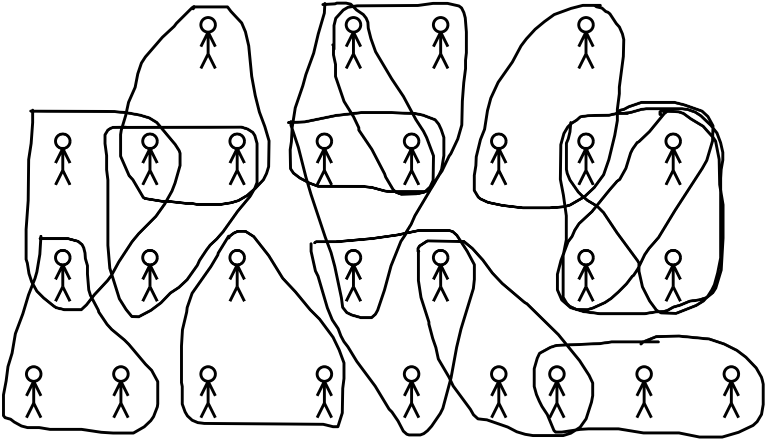

Choose who sits together

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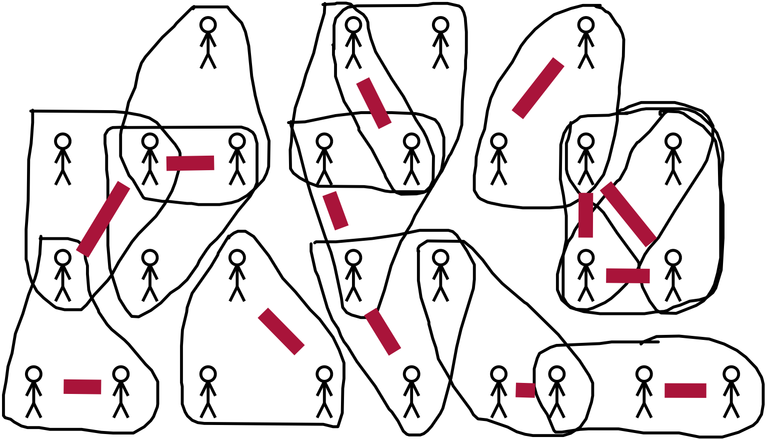

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A formal definition of GMSNP

Given: finite relational structure $\mathbb{A}$ (e.g., graph or ternary "classroom" relation)

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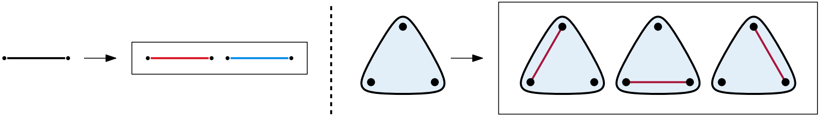

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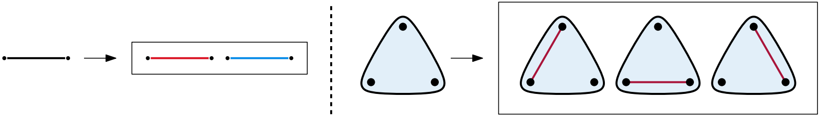
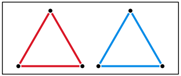
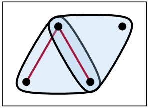


s.t. $\mathbb{A}^{\text{col}}$ is $\mathcal{F}$-free, i.e., for NO $\mathbb{F}$ from finite family $\mathcal{F}$, there is a homomorphism $\mathbb{F} \to \mathbb{A}^{\text{col}}$

$\mathcal{F} =$ 

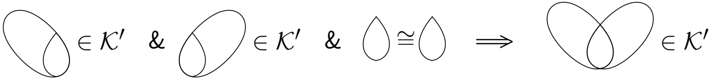
$\mathcal{F} =$ 

GMSNP is an infinite CSP

Amalgamation for GMSNP

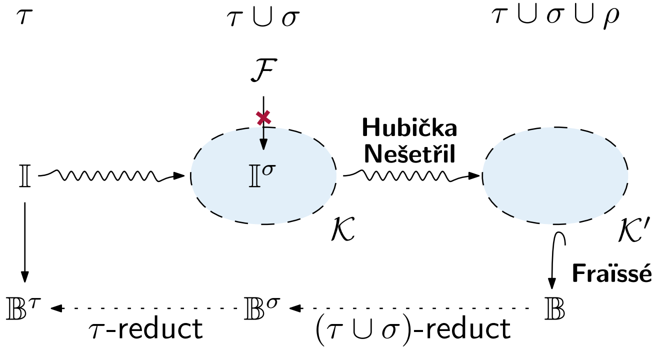
Let $\mathcal{K} :=$ all finite $\mathcal{F}$-free structures (all solutions)
Hubička, Nešetřil: there is $\mathcal{K}'$ obtained from $\mathcal{K}$ by adding finitely many new relations. $\mathcal{K}'$ is closed under taking substructures (HP) and has the amalgamation property (AP)

AP :



Fraïssé: if $\mathcal{K}'$ is closed under disjoint unions, has HP and AP, then there is homogeneous (very symmetric) countably infinite structure $\mathbb{B}$ such that $\text{Age}(\mathbb{B}) = \mathcal{K}'$

GMSNP seen as a CSP



Bodirsky, Knäuer, Starke, 2020: Input $\mathbb{I}$ has $\mathcal{F}$-free σ-expansion $\mathbb{I}^\sigma$ ($\mathbb{I} \in$ GMSNP($\mathcal{F}$)) if and only if $\mathbb{I}$ homomorphically maps to $\mathbb{B}^\tau$ ($\mathbb{I} \in$ CSP($\mathbb{B}^\tau$))

Decidability of Containment

Containment

Given: two decision problems Φ and Ψ
Task: to check whether every YES instance of Φ is a YES instance of Ψ, denoted $\Phi \subseteq \Psi$

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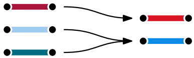
Undecidable: Datalog **(Shmueli, 1993)**, FO **(Trakhtenbrot, 1950)**

Decidable: finite-domain CSP **(obviously)** and MMSNP **(Feder, Vardi, 1998)**

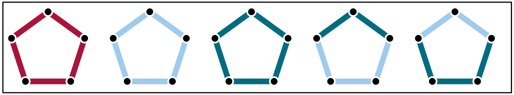
Bienvenu, ten Cate, Lutz, Wolter, 2014
Bourhis, Lutz, 2017: is containment decidable for GMSNP?

Recoloring for GMSNP

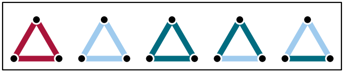
r: {colors of Φ} → {colors of Ψ} is a **recoloring** from Φ to Ψ



if the preimage $r^{-1}(\mathcal{F}_\Psi)$ has no $\mathcal{F}_\Phi$-free structures



$\mathcal{F}_\Phi$



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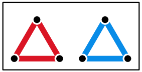


$\mathcal{F}_\Psi$

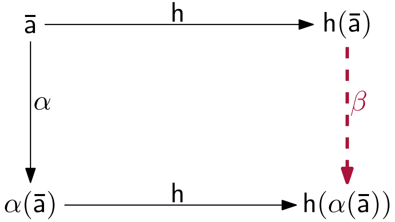
recoloring ⇒ containment

Canonical mappings

$h\colon \mathbb{A} \to \mathbb{B}$ is **canonical** w.r.t. $\mathrm{Aut}(\mathbb{A})$ and $\mathrm{Aut}(\mathbb{B})$

if for every n and every $\bar{a} \in A^n$ and every $\alpha \in \mathrm{Aut}(\mathbb{A})$

there is $\beta \in \mathrm{Aut}(\mathbb{B})$ s.t.



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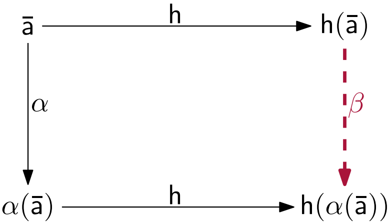

h sends n-colors of $\mathbb{A}$ to n-colors of $\mathbb{B}$!

Containment ⇒ recoloring

$$\Phi \subseteq \Psi \quad \Longrightarrow \quad \mathrm{CSP}(\mathbb{B}^{\tau}_{\Phi}) \subseteq \mathrm{CSP}(\mathbb{B}^{\tau}_{\Psi}) \quad \Longrightarrow \quad \exists h\colon \mathbb{B}^{\tau}_{\Phi} \to \mathbb{B}^{\tau}_{\Psi}$$

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$\mathbb{B}^\tau_\Phi$ has a homogeneous Ramsey expansion $\mathbb{B}_\Phi$ $\Longrightarrow$ h can be made canonical w.r.t. $\mathrm{Aut}(\mathbb{B}_\Phi)$ and $\mathrm{Aut}(\mathbb{B}_\Psi)$

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$$\{\text{colors of } \Phi'\} \xleftrightarrow{\text{bij}} \{\text{orbits of } \tau\text{-tuples in } \mathbb{B}_{\Phi'}\}$$

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Time Complexity

For GMSNP sentences Φ, Ψ

- $\mathsf{ht}(\Phi, \Psi) :=$ total number of relation symbols in Φ and Ψ
- $\mathsf{lh}(\Phi, \Psi) :=$ total number of clauses in Φ and Ψ
- $\mathsf{wd}(\Phi, \Psi) :=$ max size of clause in Φ and Ψ
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B., Pinsker, Rydval: $\Phi \subseteq \Psi$ is decidable in 2NEXPTIME w.r.t. $\mathrm{ht}(\Phi, \Psi), \mathrm{lh}(\Phi, \Psi), \mathrm{wd}(\Phi, \Psi), \mathrm{ar}(\Phi, \Psi)$

Bourhis, Lutz: $\Phi \subseteq \Psi$ is 2NEXPTIME-hard

Conclusion: $\Phi \subseteq \Psi$ is 2NEXPTIME-complete!

Conclusion

- For GMSNP, $\Phi \subseteq \Psi \Leftrightarrow \Phi' \xrightarrow{\text{rec}} \Psi'$
- Checking containment is 2NEXPTIME-complete

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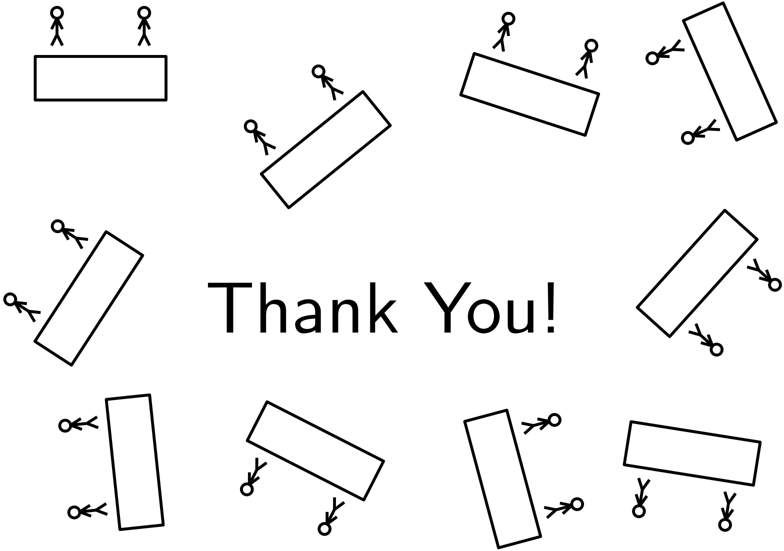
Future work

- Extend decidability of containment on larger classes
- Prove decidability for GMSNP without infinite CSP
- Characterize FO-rewritability of GMSNP problems
- Characterize complexity of GMSNP problems
- Study approximation (promise) GMSNP



Thank You!

Complexity of containment

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$\Phi \rightsquigarrow \Phi'$ for which "containment = recoloring", has size:

$$\mathsf{wd}(\Phi') := \mathsf{O}\big(\mathsf{wd}(\Phi)\big)$$
$$\mathsf{ar}(\Phi') := \mathsf{O}\big(\mathsf{ar}(\Phi) + \mathsf{wd}(\Phi)\big)$$
$$\mathsf{ht}(\Phi') := \mathsf{O}\big(\mathsf{ht}(\Phi) + \mathsf{lh}(\Phi) \cdot 2^{\mathsf{wd}(\Phi)}\big)$$
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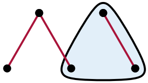
$\Phi \xrightarrow{\text{rec}} \Psi$ can be nondeterministically tested in time

$$\mathrm{O}\Big(\mathrm{lh}(\Phi, \Psi) \cdot 2^{\mathrm{wd}(\Phi,\Psi)\cdot \mathrm{ht}(\Phi,\Psi)\cdot 2^{\mathrm{ar}(\Phi,\Psi)}}\Big)$$

Why such name?

Why "guarded" and why "monotone"?

- *Guarded* – in every $\mathbb{F} \in \mathcal{F}$, "colors" are defined *within* original relational tuples



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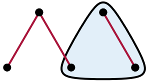
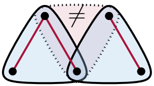


$\notin \mathcal{F}$

- *Monotone* – $\mathbb{A}^{\mathsf{col}}$ must be $\mathcal{F}$-free *homomorphism*-wise (not embedding, full homomorphism, etc.)



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Chronology

History of GMSNP

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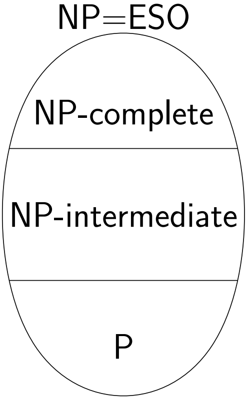
- **B., Pinsker, Rydval (2025):** containment for GMSNP is decidable

Dichotomy question

The dichotomy question



Ladner: If P $\neq$ NP, then NP has problems that are neither in P nor NP-complete.
Fagin: The problems in NP are precisely those that are described by sentences in Existential Second-Order logic (ESO).

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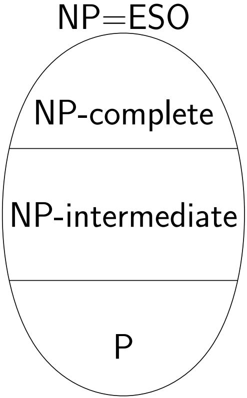

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Question

For a given logic $\mathcal{L} \subset$ ESO, is $\mathcal{L}$ a subset of (P $\cup$ NP-complete)?

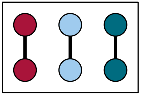
The dichotomy question

finite CSP $\subset$ MMSNP $\subset$ GMSNP $\subset$ CSPs of first-order reducts of finitely bounded homogeneous structures $\subset$ Monotone SNP

Given: finite relational structure

Task: assign to every vertex one of the several colors

such that the result is $\mathcal{F}$-free

$\mathcal{F} =$ 

The dichotomy question

finite CSP $\subset$ MMSNP $\subset$ GMSNP $\subset$ CSPs of first-order reducts of finitely bounded homogeneous structures $\subset$ Monotone SNP

Feder, Vardi: Every problem in MMSNP is P-time equivalent to CSP with finite domain

Zhuk, Bulatov: Finite CSPs have dichotomy that is characterized by algebraic properties of the template.

The dichotomy question

$$\text{finite CSP} \subset \text{MMSNP} \subset \text{GMSNP} \subset \text{CSPs of first-order reducts of finitely bounded homogeneous structures} \subset \text{Monotone SNP}$$

$\mathbb{A}$ is **homogeneous** if every isomorphism between its finite substructures extends to an automorphism of $\mathbb{A}$.

$\mathbb{A}$ is **finitely bounded** if for some finite family $\mathcal{F}$

$\forall\, \mathbb{B}$ finite $(\mathbb{B} \subset \mathbb{A} \Leftrightarrow \forall\, \mathbb{F} \in \mathcal{F} \;\; \mathbb{F} \not\to \mathbb{B})$ (Age($\mathbb{A}$) is $\mathcal{F}$-free)

The dichotomy question

finite CSP	$\subset$	MMSNP	$\subset$	GMSNP	$\subset$	CSPs of first-order reducts of finitely bounded homogeneous structures	$\subset$	Monotone SNP

$\mathbb{B}$ is a **first-order reduct** of $\mathbb{A}$ if $\mathbb{B}$ has the same domain as $\mathbb{A}$ and if every relation of $\mathbb{B}$ is first-order definable in $\mathbb{A}$.

Conjecture (Bodirsky, Pinsker): CSPs of such structures have dichotomy characterized by algebraic properties of the template.

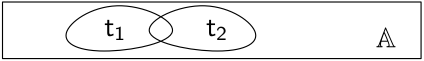
The dichotomy question

finite CSP ⊂ MMSNP ⊂ GMSNP ⊂ CSPs of first-order reducts of finitely bounded homogeneous structures ⊂ Monotone SNP

Given: finite relational structure $\mathbb{A}$

Task: assign a color to each k-element subset of $\mathbb{A}$ (k is fixed)

s.t. the colors assigned to intersecting subsets are compatible



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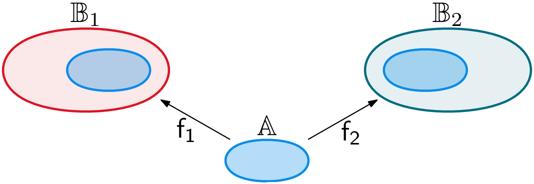
finite CSP $\subset$ MMSNP $\subset$ GMSNP $\subset$ CSPs of first-order reducts of finitely bounded homogeneous structures $\subset$ Monotone SNP

Feder, Vardi: Every problem in NP is P-time equivalent to a problem in Monotone SNP

Amalgamation and Ramsey properties

Amalgamation Property (AP)

Class $\mathcal{K}$ has **amalgamation** property if for all $\mathbb{A}, \mathbb{B}_1, \mathbb{B}_2 \in \mathcal{K}$ and all $f_1 : \mathbb{A} \hookrightarrow \mathbb{B}_1$, $f_2 : \mathbb{A} \hookrightarrow \mathbb{B}_2$ there is $\mathbb{C} \in \mathcal{K}$ and $g_1 : \mathbb{B}_1 \hookrightarrow \mathbb{C}$ and $g_2 : \mathbb{B}_2 \hookrightarrow \mathbb{C}$ such that $g_1 \circ f_1 = g_2 \circ f_2$



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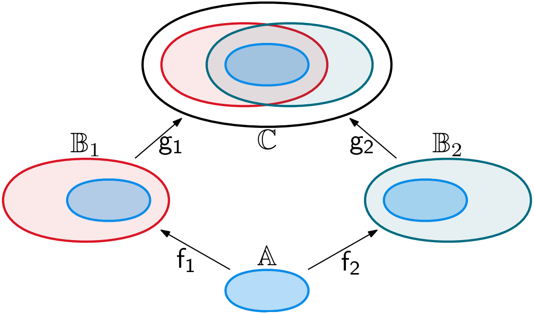
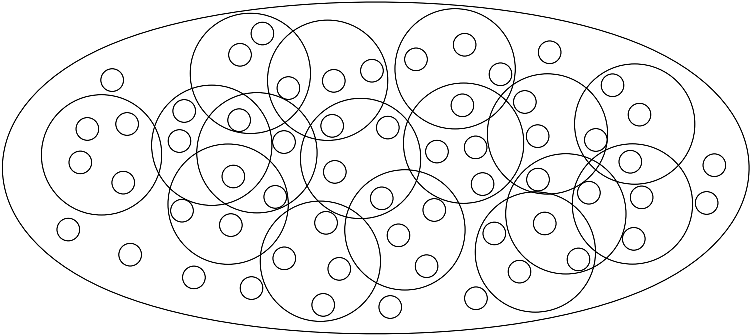


Ramsey property

Class $\mathcal{K}$ is **Ramsey** if for all $\mathbb{A}, \mathbb{B} \in \mathcal{K}$ and all $\mathsf{n} \in \mathbb{N}$ there is $\mathbb{C} \in \mathcal{K}$ s.t. for all $\chi\colon \binom{\mathbb{C}}{\mathbb{A}} \to [\mathsf{n}]$ there is $\mathbb{B}_0 \in \binom{\mathbb{C}}{\mathbb{B}}$ s.t. χ is constant on $\binom{\mathbb{B}_0}{\mathbb{A}}$



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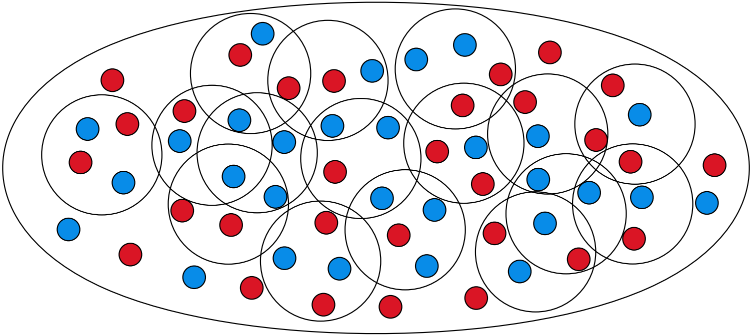

Ramsey property

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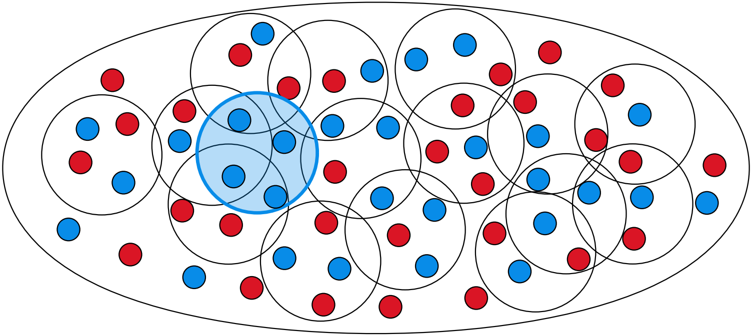
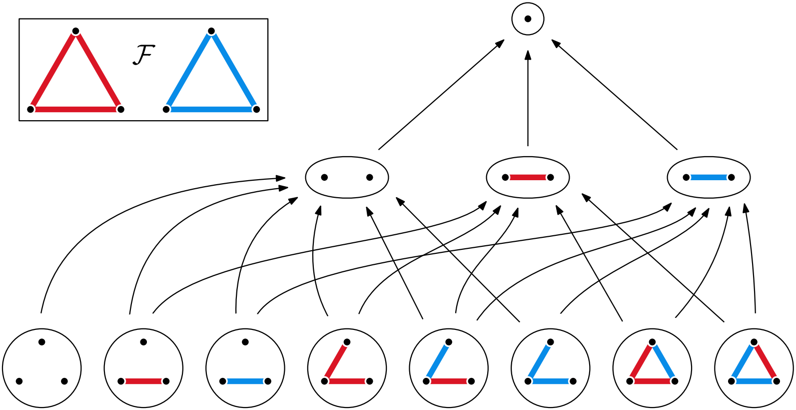


Finitely bounded homogeneous CSPs

No-monochromatic-triangle as finite CSP



No-monochromatic-triangle as finite CSP