Homework 2 - Is the first derivative a generator?

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Consider operators A_i, where $A_i f = f'$, on various subspaces of $C([0,1])$ with various domains:

1. Let

$$X_1 := C([0,1]), \quad D(A_1) := C^1([0,1]).$$

Show that $\lambda - A_1$ is never injective (domain is too big), and therefore A_1 is not the generator a C_0-semigroup (why?).

2. Let

$$X_2 := C([0,1]), \quad D(A_2) := \{f \in C^1([0,1]), f'(1) = 0\}.$$

Show with help of the Lumer–Phillips theorem that A_2 is the generator of a C_0-semigroup.

3. Let

$$X_3 := C([0,1]), \quad D(A_3) := \{f \in C^1([0,1]), f(1) = 0\}.$$

Show that A_3 is not densely defined (domain is too small), and therefore A_3 is not the generator of a C_0-semigroup. Show that the remaining assumptions of the Hille–Yosida theorem are satisfied (closedness, condition on the resolvent set and resolvent estimate).

4. Show that A_3 generates a C_0-semigroupu on the closure of its domain, i.e.

$$X_4 := \{f \in C([0,1]), f(1) = 0\}, \quad D(A_4) := \{f \in C^1([0,1]), f(1) = 0, f'(1) = 0\}$$

is the generator of a C_0-semigroup.

Find the semigroups generated by A_2, A_4.