Homework 3 - self-adjoint semigroups

submit before April 4

Let $H = L^2(0,1)$, $Af = f''$, $D(A) = \{f \in H^2(0,1) : \ f(0) = f(1) = 0\}$.

1. Show that A is symmetric.

2. Show that A is dissipative.

3. Find a solution to $y'' - y = g$ for a given $g \in L^2$ and conclude that A is self-adjoint.

4. Show that A has a compact resolvent (use embeddings of Sobolev spaces)

5. Find the spectrum and eigenvectors of A

6. Find U and the sequences α_n and ϕ_n from Spectral theorem II.