Homework 4 - Regularity of semigroups

submit before April 11

Let T be the shift semigroup $(T(t)f)(s) = f(s+t)$ on $C_0([0,+\infty), \mathrm{C})$ and A its generator $Af = f'$, $D(A) = \{f \in C^1 : f, f' \in C_0\}$.

1. Show that T is not norm-continuous at any $t > 0$ (norm continuous means $\lim_{h\to 0} \|T(t+h) - T(t)\| = 0$ in the operator norm).

2. Find the spectrum of A. [You can compute the eigenvalues and then compute $\|T(t)\|$ and use Proposition 4]

Let S be the shift semigroup on $X = \{f \in C([0,1]) : f(1) = 0\}$ and B its generator $Bf = f'$, $D(B) = \{f \in C^1([0,1]) : f(1) = f'(1) = 0\}$. [Find a formula for $S(t)$!!!]

3. Find the spectrum of B. [Compute $\|S(t)\|$ for $t > 1$ and apply Proposition 4. You need to have a formula for $S(t)$.]

4. Show that S is not analytic. [It is enough to show that it is not norm-continuous.]

5. Show that S is differentiable on $(1,+\infty)$. [If you have a formula for $S(t)$ then it is immediate.]

Let R be a C_0-semigroup with the generator C and $t_0 > 0$. Let $R(t_0)x \in D(C)$ for all $x \in X$.

6. Show that $S(nt_0)x \in D(C^n)$ and $t \mapsto S(t)x$ is n-times differentiable on $(nt_0, +\infty)$ for all $x \in X$. [By induction...]