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CSP DICHOTOMY FOR SPECIAL TRIADS

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1. Added after posting

This section corrects two mistakes in the article. The first mistake concerns the situation when a special triad is not a core: the characterization given in Lemma 4.3 is incomplete as it can happen that all the paths $\mathbb{P}_1, \ldots, \mathbb{P}_6$ are mapped to one of the paths $\mathbb{P}_4, \ldots, \mathbb{P}_6$. Unfortunately this is the case in the example of the special triad in Figure 1. To obtain a special triad which is NP-complete, we replace the path $\mathbb{P}_4$ by $\mathbb{P}_2$ and paths $\mathbb{P}_5$, $\mathbb{P}_6$ both by $\mathbb{P}_1$ (the new triad has 39 vertices). This mistake doesn't influence the dichotomy result for special triads, as it is well known that CSP($\mathbb{G}$) is tractable for any oriented path $\mathbb{G}$.

The second mistake is in the classification given in Theorem 3.4. To correct it we need an auxiliary notation: for oriented paths $\mathbb{P}_1, \ldots, \mathbb{P}_k$ with initial vertices $i_1, \ldots, i_k$ let $c(\mathbb{P}_1 \times \cdots \times \mathbb{P}_k)$ be the connectivity component of the digraph $\mathbb{P}_1 \times \cdots \times \mathbb{P}_k$ containing the vertex $(i_1, \ldots, i_k)$. The right statement of Theorem 3.4. is obtained by replacing all products with their connectivity components:

Theorem 3.4. *For every special triad $\mathbb{G}$, CSP($\mathbb{G}$) is either tractable or NP-complete.*

More specifically, let $\mathbb{G}$ be the special triad given by paths $\mathbb{P}_1, \ldots, \mathbb{P}_6$.

1. *If there exist $i, j \in \{1, 2, 3\}$, $i \neq j$ and a homomorphism $c(\mathbb{P}_{i+3} \times \mathbb{P}_j) \to \mathbb{P}_i$, then $\mathbb{G}$ admits a compatible totally symmetric idempotent operation of any arity.*
2. *If there exist $i, j, k \in \{1, 2, 3\}$ pairwise distinct and homomorphisms $c(\mathbb{P}_{i+3} \times \mathbb{P}_{j+3} \times \mathbb{P}_k) \to \mathbb{P}_i$, $c(\mathbb{P}_{i+3} \times \mathbb{P}_j \times \mathbb{P}_{k+3}) \to \mathbb{P}_i$, $c(\mathbb{P}_{i+3} \times \mathbb{P}_j \times \mathbb{P}_k) \to \mathbb{P}_i$, then $\mathbb{G}$ admits a compatible majority operation.*
3. *If $\mathbb{G}$ is not a core, then either one of the cases (1), (2) can be applied, or the core of $\mathbb{G}$ is an oriented path.*
4. *Otherwise, CSP($\mathbb{G}$) is NP-complete.*

The proof of the theorem remains the same except for replacing all products of oriented paths with their connectivity component containing the initial vertex. This change is necessary because in their original formulation Lemmas 5.5 and 5.7 might not be true—for example in Lemma 5.5. the nonexistence of a homomorphism from $\mathbb{P}_4 \times \mathbb{P}_2$ to $\mathbb{P}_1$ doesn't necessarily imply that $0 \not\to 1$ in $\mathbb{G}^{\{2,4\}}$, since a homomorphism can map the other connectivity components of $\mathbb{P}_4 \times \mathbb{P}_2$ outside the path $\mathbb{P}_1$.

Because of this change we also need to adjust the proofs of Lemmas 4.1 and 4.2. In Lemma 4.1 when defining the homomorphism h from $\mathbb{H}$ to $\mathbb{G}$ we need to distinguish one more case:

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(0) If all the vertices in R have the same level and are in a connectivity component of $\mathbb{H}$ other than $\{0\}$ then we put $h(R)$ to be the smallest vertex in R in the ordering

$$\xrightarrow{\mathbb{P}_1} \quad \xrightarrow{\mathbb{P}_2} \quad \xrightarrow{\mathbb{P}_3} \quad \xrightarrow{\mathbb{P}_4} \quad \xrightarrow{\mathbb{P}_5} \quad \xrightarrow{\mathbb{P}_6}$$

Note that in this case $R \cap \{0, 1, 2, 3, 4, 5, 6\} = \emptyset$.

In the proof of Lemma 4.2. we also need to add one more case:

(0) If a, b, c have the same level, doesn't lie on an oriented subpath of $\mathbb{G}$ and (a, b, c) is in a connectivity component of $\mathbb{G}^3$ other than the vertex $(0, 0, 0)$, then we put $m(a, b, c) = a$.

We wish to thank Jakub Bulín for carefully reading the article and finding the above two mistakes.