CSP lecture 21/22 winter semester – Problem Set 1

Consider the following relations on $\{0, 1\}$:

- $C_i := \{i\}$, for $i \in \{0, 1\}$
- $R := \{(0, 0), (1, 1)\}$
- $N := \{(0, 1), (1, 0)\}$
- $S_{ij} := \{0, 1\}^2 \setminus \{(i, j)\}$, for $i, j \in \{0, 1\}$
- $H := \{0, 1\}^3 \setminus \{(1, 1, 0)\}$
- $G_1 := \{(0, 0, 0), (0, 1, 1), (1, 0, 1), (1, 1, 0)\}$, $G_2 := \{(0, 0, 1), (0, 1, 0), (1, 0, 0), (1, 1, 1)\}$

Problem 0. Prove that the definitions of CSP($\mathbb{A}$) (satisfiability of a list of constraints and homomorphism problem) are equivalent.

Problem 1. Find a polynomial–time algorithm for CSP($\mathbb{A}$), where

1. $\mathbb{A} = (\{0, 1\}; R)$
2. $\mathbb{A} = (\{0, 1\}; R, C_0, C_1)$
3. $\mathbb{A} = (\{0, 1\}; S_{10})$
4. $\mathbb{A} = (\{0, 1\}; S_{10}, C_0, C_1)$
5. $\mathbb{A} = (\{0, 1\}; S_{01}, S_{10}, C_0, C_1)$
6. $\mathbb{A} = (\{0, 1\}; N)$
7. $\mathbb{A} = (\{0, 1\}; R, N, C_0, C_1)$
8. $\mathbb{A} = (\{0, 1\}; R, N, C_0, C_1, S_{00}, S_{01}, S_{10}, S_{11})$
9. $\mathbb{A} = (\{0, 1\};$ all unary and binary relations$)$

Problem 2. Find a polynomial–time algorithm for CSP($\{0, 1\}; H, C_0, C_1$).

Problem 3. Find a polynomial–time algorithm for CSP($\{0, 1\}; C_0, C_1, G_1, G_2$).

Problem 4. Find a polynomial–time algorithm for CSP($\mathbb{Q}; <$).

Problem 5. Prove that CSP($\mathbb{Q}; <$) $\neq$ CSP($\mathbb{A}$), for every finite relational structure $\mathbb{A} = (A; R)$.