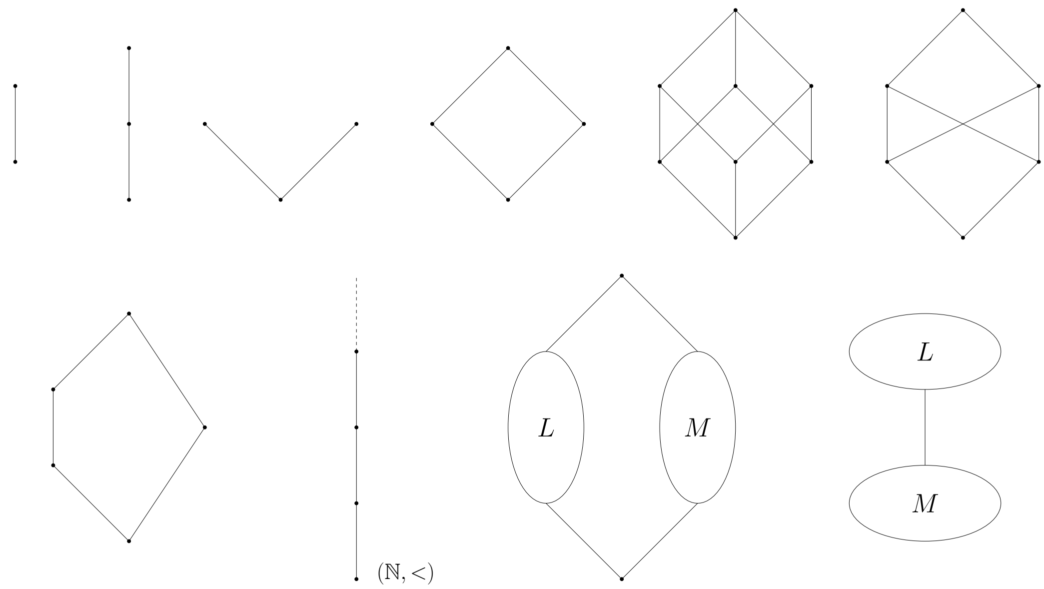
# Universal Algebra Exercises - Sheet 1

Recall the definitions of (semi)lattices and (semi)lattice ordered sets.

**Exercise 1.** Which of the following Hasse Diagrams represent (semi)lattices, given that $L$ and $M$ are lattices?




**Exercise 2.** Find the correspondence between lattices and lattice ordered sets:

(i) For a given semilattice $(S, \wedge)$ find a natural definition of a semilattice order on S, and vice-versa.

(ii) Conclude that there is a 1-to-1 correspondence between semilattices and semilattice ordered sets.

(iii) Prove that there is a 1-to-1 correspondence between lattices and lattice ordered sets.

**Exercise 3.** Show that every homomorphism of lattices is order preserving. What about the converse?

**Exercise 4.** Choose your favorite (finite) group and draw the poset of its normal subgroups, ordered by inclusion of their underlying sets.

**Exercise 5.** Given a group $G$, prove that the poset of its normal subgroups is a lattice (ordered set). What are the operations meet and join?

**Exercise 6.** Let $(P, \leq)$ be a poset. Show that there is a linear order $\leq'$ on $P$ such that $p \leq q \implies p \leq' q$ for all $p, q \in P$.

**Exercise 7** (*)**.** Let $(P, \leq)$ be a poset, $(\mathcal{U}_P, \subseteq)$ the poset of its upsets and $(\mathcal{D}_P, \subseteq)$ the poset of its downsets.

(i) Find an injective order preserving map $(P, \leq) \to (\mathcal{D}_P, \subseteq)$

(ii) Show that there is no surjective order preserving map $(P, \leq) \to (\mathcal{D}_P, \subseteq)$

(iii) Conclude that for any set $X$, there is no surjective map $X \to 2^X$

**Exercise 8.** Prove or disprove that for every poset $(P, \leq)$ we have

$$(\mathcal{D}_P, \subseteq) \cong (\mathcal{U}_P, \supseteq)$$