Universal Algebra 2 - Exercises 2

Exercise 2.1. Find a digraph that is locally confluent, but not confluent.

Exercise 2.2. Let $\mathcal{E} = \{(x \cdot y) \cdot z \approx x \cdot (y \cdot z)\}$ be the theory of semigroups. Show that $D(\mathcal{E})$ is convergent and conclude that the term rewriting algorithm works in this case.

Exercise 2.3. Consider $\mathcal{E} = \{f(f(x)) \approx g(x)\}$.

(i). Show that $D(\mathcal{E})$ is not convergent and try to understand why the term rewriting algorithm doesn't work.

Use the Knuth-Bendix algorithm to find a convergent rewriting system equivalent to $\mathcal{E}$.

(ii). Find a suitable reduction order.

(iii). Find a critical pair of $\mathcal{E}$ and check local confluence.

(iv). What are the normal forms of terms?

Exercise 2.4. Consider $\mathcal{E} = \{(x \cdot y) \cdot z \approx x \cdot (y \cdot z),\ x \cdot x \approx x\}$.

(i). Use the Knuth-Bendix algorithm to expand $\mathcal{E}$ by at least two equations.

(ii). Show that the algorithm enters an infinite loop.

Exercise 2.5. Find a convergent system for the theory of groups. (Hint: think of a normal form and don't use Knuth-Bendix.)

Exercise 2.6. Show that $D(\mathcal{E})$ is not convergent for any axiomatization $\mathcal{E}$ of the variety of commutative rings (Hint: $x + y \approx y + x$). Think of ways to modify the definitions of convergence, normal forms and reductions and describe the resulting term rewriting algorithm.