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Let t be an n-ary operation on A, B a set, $f: A \to B$ and $g: B \to A$ mappings. We define an n-ary operation $t^{(f,g)}$ on B by $t^{(f,g)}(x_1, \dots, x_n) := f(t(g(x_1), \dots, g(x_n)))$

("a <u>reflection</u> of t")

For clones $\mathcal{A}, \mathcal{B}$ write $\mathcal{B} \in ER(\mathcal{A})$ if there exist $f: A \to B$ and $g: B \to A$ such that

$$\{ t^{(f,g)} ; t \in \mathcal{A} \} \subseteq \mathcal{B}.$$

3.1 Prove that the following are equivalent for any clones $\mathcal{A}, \mathcal{B}$ on finite sets:

(i) $\mathcal{A} \overset{\min}{\leq} \mathcal{B}$

(ii) $\mathcal{B} \in ERP(\mathcal{A})$

(Hint: use an argument similar to the proof of $\leq \Leftrightarrow$ EHSP)

~~3.2~~

A clone $\mathcal{A}$ on a finite set A is called a <u>core</u> if each unary member of $\mathcal{A}$ is a bijection

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(3.2) a) Prove that for any core clone $\mathcal{A}$ on a finite set and a unary member f, $f^{-1} \in \mathcal{A}$

b) Prove that for any clone $\mathcal{A}$ on a finite set there is a core clone $\mathcal{B}$ on a finite set such that $\mathcal{A} \overset{min}{\sim} \mathcal{B}$

(Hint for (ii) Consider the image S of a unary $f \in \mathcal{A}'$ (suitably chosen) and define $\mathcal{B}$ on S as restrictions of those members of $\mathcal{A}$ that preserve S)

(3.3) Prove that for any core clone $\mathcal{A}$, $\mathcal{A} \overset{min}{\sim} \mathcal{A}^{(id)}$, where

$$\mathcal{A}^{(id)} = \{ f \in \mathcal{A} ; f \text{ is idempotent} \}$$

(Hint: Modify each operation in $\mathcal{A}$ (by composing it with a unary operation) to make it idempotent and prove that this modification is a minion homo.)