UA II
Practicals
16.05

1) Show that

$(\{0,1\}, \vee)$ does <u>not</u> have few subpowers.

A k-ary <u>near unamity (NU)</u> operation t is defined by:

$$x \approx t(yx\dots x) \approx t(xyx\dots x) \approx \dots \approx t(x\dots xy)$$

2) Let $t: A^k \to A$ be NU on a ~~finite~~ set A. Show that

$$\forall R \in \mathrm{Inv}(t): \quad R = \bigwedge_{|I|<k} \underbrace{pr_I R}_{\text{projection of } R \text{ to coordinates in } I=\{i_1, i_2 \dots i_m\}} \quad (*)$$

Thus NU $\Rightarrow$ few subpowers

(Hint: Induction on arity(R))

3*) <u>Theorem (Baker, Pixley '75)</u>

Let $\underline{A}$ be a finite algebra such that $\forall R \in \mathrm{Inv}(\underline{A})$ $(*)$ holds.

Show that then $\underline{A}$ has a k-ary NU term.

4) Let $\underline{A} = (\{a,b,c\}, g)$ such that

- $g: A^3 \to A$ is symmetric, idempotent
- $(\{a,b\}, g) \cong (\{0,1\}, maj)$
- $(\{a,c\}, g) \cong (\{0,1\}, x+y+z)$
- $g(bbc) = g(abc) = c, \quad g(bcc) = a$

Show that $\underline{A}$ has no Mal'tsev or NU term, *) but $\underline{A}$ has few subpowers.