

UA Exercises 13

5th Jan 2023

① Prove that an equivalence α is a congruence iff α is invariant under every unary polynomial

② Find all clones on $\{0,1\}$ containing the constant operations.

③ \underline{A} minimal algebra, $|A| \geq 3$, $f(x, \bar{y}) \in \text{Pol}(\underline{A})$.
 If $\bar{c}, \bar{d} \in A$, then $x \mapsto f(x, \bar{c})$ is a permutation not st-p
 iff $x \mapsto f(x, \bar{d})$ is

deduce: \underline{A} minimal, $|A| \geq 3$, $f(x, \bar{y}) \in \text{Pol}(\underline{A})$, $\bar{a}, \bar{b} \in A^n, c, d \in A$
 $f(c, \bar{a}) = f(d, \bar{a}) \Rightarrow f(c, \bar{b}) = f(d, \bar{b})$

④ \underline{A} algebra, U neighborhood with witness e

- $\underline{A}|_U = \{ef \mid f \text{ polynomial of } \underline{A}\}$
- \underline{A} has Mal'cev/majority, then so does $\underline{A}|_U$
- \underline{A} Mal'cev/majority, then $\text{typ } \underline{A} \subseteq \{2,3\} / \text{typ } \underline{A} \subseteq \{3,4\}$

⑤ \underline{A} simple

- each minimal set is a neighborhood
- $a \neq b, U$ minimal $\Rightarrow \exists f \in \text{Pol}_1(\underline{A}) \quad f(a) = U, f(b) \neq f(a)$
- $\forall a, b \in A \quad \exists U_1, \dots, U_n$ minimal $a \in U_1, U_1 \cap U_2 \neq \emptyset, U_2 \cap U_3 \neq \emptyset, \dots, b \in U_n$
- $\forall U, V$ minimal $\exists f, g \in \text{Pol}_1(\underline{A}) \quad f(U) = V, g(V) = U$
 $g \circ f \upharpoonright_U = \text{id}_U, f \circ g \upharpoonright_V = \text{id}_V$