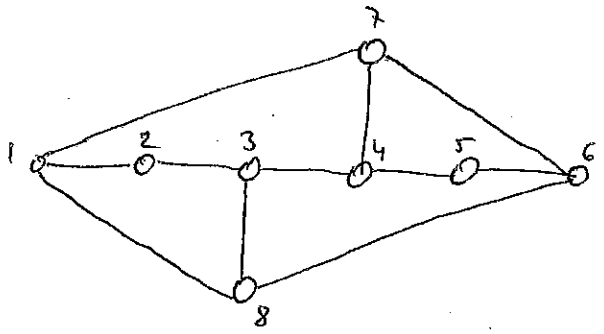
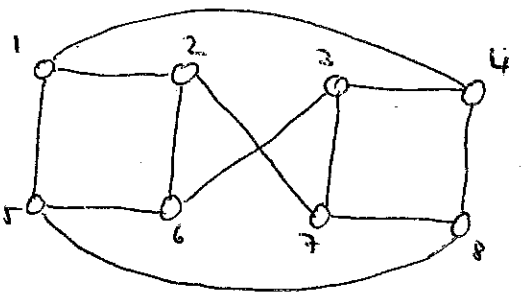
Homework 2

(2.1) Let G be the graph with vertex set $\{1, 2, \ldots, 15\}$ in which i and j are adjacent iff their greatest common factor exceeds 1. Count the components of G and determine the maximum length of a path in G.

(2.2) In each graph below, find a bipartite subgraph with the maximum number of edges. Prove that this is the maximum, and determine whether this is the only bipartite subgraph with this many edges.





(2.3) Prove that a graph G is bipartite if and only if every subgraph H of G has an independent set consisting of at least half of $V(H)$.