Homework 5

(5.1) A saturated hydrocarbon is a molecule formed from k carbon atoms and ℓ hydrogen atoms such that each carbon atom is in four bonds, each hydrogen atom is in one bond, and no sequence of bonds forms a cycle of atoms. Prove that $\ell = 2k+2$.

(5.2) Prove that the center of a graph can be disconnected and can have components arbitrarily far apart by constructing a graph where the center consists of two vertices and the distance between these two vertices is k. (construct such a graph for each k)

(5.3) Let T be a tree. Prove that the vertices of T all have odd degree if and only if for all $e \in E(T)$, both components of $T-e$ have odd number of vertices.