Homework 7

(7.1) Prove that every tree has at most one perfect matching.

(Hint: consider the symmetric difference of two perfect matchings.)

(7.2) A _line_ in a $0,1$-matrix is a row or a column, and two entries are _independent_ if they do not lie in a common line. Prove that the maximum number of pairwise independent 1s in a $0,1$-matrix equals the minimum number of lines that together contain all the 1s.

(7.3.) Let $\underline{A} = (A_1, \ldots A_m)$ be a collection of subsets of a set Y. A _system of distinct representatives_ (SDR) for $\underline{A}$ is a set of distinct elements $a_1, \ldots, a_m$ in Y such that $a_i \in A_i$ (for all $i = 1, 2, \ldots, m$). Prove that $\underline{A}$ has an SDR if and only if $\left| \bigcup_{i \in S} A_i \right| \geq |S|$ for every $S \subseteq \{1, 2, \ldots, m\}$.

(Hint: Transform this to a graph problem)