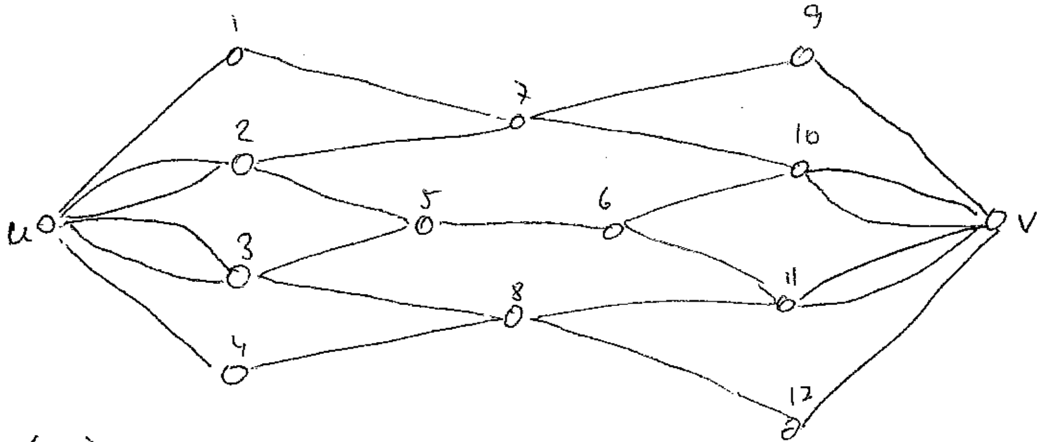
Homework 9 - Solutions

(9.1)



$\kappa(u,v) \le 3$ since by removing vertices 5, 7, 8 u, v become disconnected

$\lambda(u,v) \ge 3$ since u179v, u25610v, u4812v are internally disjoint u,v-paths

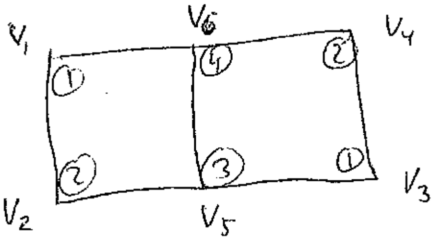
$\rightarrow$ $\boxed{\kappa(u,v)\ (= \lambda(u,v)) = 3}$

$\kappa'(u,v) \le 5$ since removal of edges 17, 27, 56, 38, 48 disconnects u and v

$\lambda'(u,v) \ge 5$ since u179v, u~2710~v, u~25610~v, u3811v, u4812v are edge-disjoint u,v-paths

Therefore $\boxed{\kappa'(u,v) = 5}$ $(= \lambda'(u,v))$

(9.2)

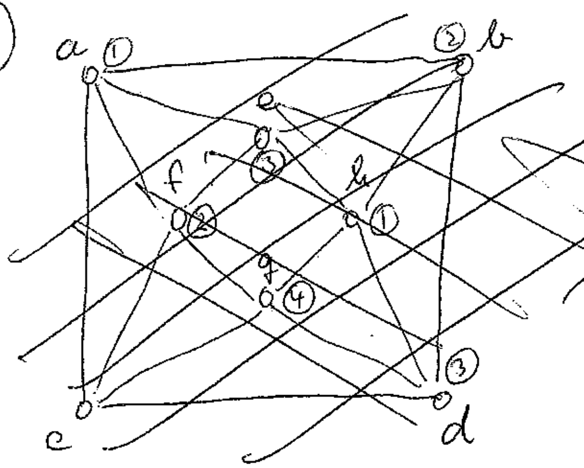


with the vertex ordering $v_1, v_2, \dots, v_6$ the greedy algorithm gives the coloring

$v_1 \to 1,\ v_2 \to 2,\ v_3 \to 1,\ v_4 \to 2,\ v_5 \to 3,\ v_6 \to 4$

it uses $4 = \Delta(G)+1$ colors

(9.3)



$\chi(G) \le 4$ as shown on the picture

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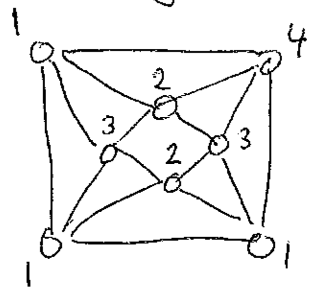
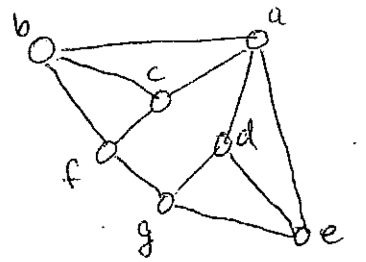
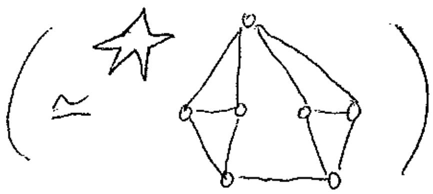
(9.3) The following coloring proves $\chi(G) \le 4$.



Let H be the following subgraph

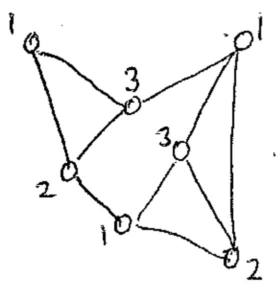




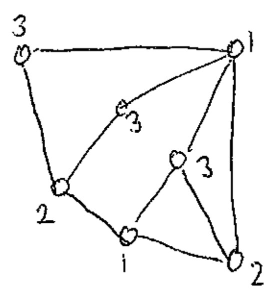
- $\chi(H) > 3$: Striving for a contradiction assume that H has a 3-coloring F. WLOG assume $F(a)=1$. Then $F(d)=2$ & $F(e)=3$ or vice versa, in any case $F(g)=1$. Similarly $F(f)=1$. But then fg is a monochromatic edge, a contradiction.
  Now $4 \ge \chi(G) \ge \chi(H) > 3$, thus $\chi(G) = \chi(H) = 4$ //

- H is 4-critical: The following pictures show proper 3-colorings of the subgraphs obtained by removing edges ab, bc, bf, fg. The other cases are symmetric (see ☆)

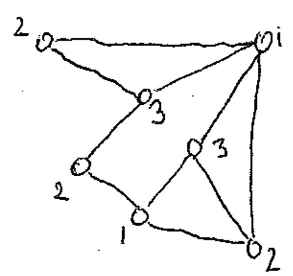
ab



bc



bf



fg