Recommended Problems 5

(5.1) Let $d_1, \ldots, d_n$ be positive integers, with $n \geq 2$. Prove that there exists a tree with vertex degrees $d_1, \ldots, d_n$ if and only if $\sum d_i = 2n - 2$.

(5.2) Let T, T' be two spanning trees of a connected graph G. For $e \in E(T) - E(T')$, prove that there is an edge $e' \in E(T') - E(T)$ such that $T' + e - e'$ and $T - e + e'$ are both spanning trees of G.

(5.3) Every tree is bipartite. Prove that every tree has a leaf in its larger partite set (in both if they have equal size).

(5.4) Let T be a tree of even order.(*) Prove that T has exactly one spanning subgraph in which every vertex has odd degree.

(*) "even order" means even number of vertices