Recommended Problems 6

6.1 Use Cayley's Formula to prove that the graph obtained from K_n by deleting an edge has $(n-2)n^{n-3}$ spanning trees. (Hint: By symmetry, each edge lies in the same number of spanning trees of K_n)

6.2 Let G_n be the graph with $2n$ vertices and $3n-2$ edges pictured below, and let $\tau(G_n)$ denote the number of spanning trees of G_n. Prove for $n>2$ that $\tau(G_n) = 4\tau(G_{n-1}) - \tau(G_{n-2})$

6.3 Assign integer weights to the edges of K_n. Let the weight of a cycle be the sum of the weights of its edges. Prove that all cycles have even weight if and only if the subgraph formed by the edges with odd weight is a spanning biclique.
(Hint: Consider the subgraph formed by the even-weight edges.)

6.4 <u>DJP algorithm</u> grows a spanning tree from a given vertex of a connected weighted graph G, iteratively adding the cheapest edge from a vertex already reached to a vertex not yet reached, ~~finni~~ finishing when all the vertices of G have been reached. (Ties are broken arbitrarily.) Prove that DJP algorithm produces a minimum-weight spanning tree of G.