Problem Set 3

Problem 1. Consider the relational structure $\mathbb{A} = (\{0,1\}; R)$, where $R = \{(a,b) : a \leq b\}$. Does there exist an instance of CSP($\mathbb{A}$) with no solution?

Problem 2. Let $\mathbb{A} = (\{0,1\}; \{0\}, \{1\}, R)$, where $R = \{(a,b) : a \leq b\}$.

(a) Does the following instance of CSP($\mathbb{A}$) have a solution?

$$(x_2, x_1) \in R \ \& \ x_3 \in \{1\} \ \& \ (x_1, x_4) \in R \ \& \ (x_1, x_3) \in R \ \& \ x_4 \in \{0\} \ \& \ (x_3, x_2) \in R$$

(b) Describe instances of CSP($\mathbb{A}$) which have a solution.

(c) Find a fast algorithm for solving CSP($\mathbb{A}$).

Problem 3. Let $\mathbb{A} = (\{0,1\}; R)$, where $R = \{(0,1), (1,0)\}$.

(a) Do the following instances of CSP($\mathbb{A}$) have a solution?

- $(x,y) \in R \ \& \ (z,y) \in R \ \& \ (z,x) \in R$
- $(x,y) \in R \ \& \ (z,y) \in R \ \& \ (z,v) \in R \ \& \ (v,x) \in R$
- $(x,y) \in R \ \& \ (y,x) \in R \ \& \ (z,y) \in R \ \& \ (y,z) \in R \ \& \ (z,v) \in R \ \& \ (v,x) \in R$

(b) Describe instances which have a solution.

(c) Find a fast algorithm for solving CSP($\mathbb{A}$).

Problem 4. Let $\mathbb{A} = (\{0,1\}; \{0\}, \{1\}, R)$, where $R = \{0,1\}^3 \setminus (1,1,0)$. Find a fast algorithm for solving CSP($\mathbb{A}$).

Problem 5. Let $\mathbb{A}$ be a relational structure such that some constant mapping is a polymorphism of $\mathbb{A}$. Prove that every instance of CSP($\mathbb{A}$) has a solution.

Problem 6. Let $\mathbb{A} = (\{0,1,2\}; \{0\}, \{1\}, \{2\}, R)$, where $R = \{(x,y,z) : x + y + z = 0\}$ (we add modulo 3). Find a fast algorithm for solving CSP($\mathbb{A}$).

Problem 7. An instance of 2-CLAUSE-CONJ is a conjunction of clauses, where each clause is a disjunction of at most two variables, possibly negated. The question is whether this boolean formula is satisfiable.

(a) Is the following instance of 2-CLAUSE-CONJ satisfiable?

$$(x \vee \neg y) \wedge z \wedge (\neg x \vee \neg z)$$

(b) Find a relational structure $\mathbb{A}$ such that CSP($\mathbb{A}$) can be viewed as 2-CLAUSE-CONJ

(c*) Find a fast algorithm for solving 2-CLAUSE-CONJ