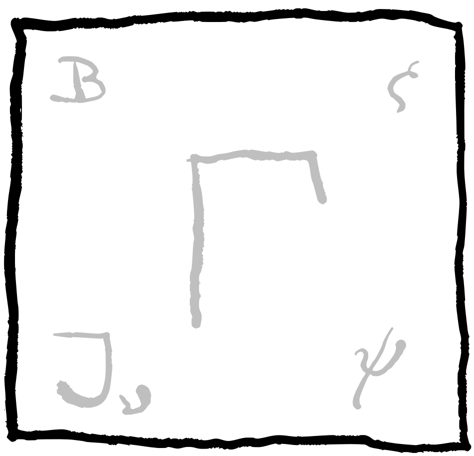



# SPECIÁLNÍ FUNKCE

# GAMMA FUNKCE

$\boxed{D}$ $\Gamma(z) := \int_0^\infty t^{z-1} e^{-t}\, dt \; ; \quad \operatorname{Re} z > 0$

$\boxed{V}$ (VLASTNOSTI $\Gamma$ FCE)

i) Speciální hodnoty : $\Gamma(n) = (n-1)! \; ; \; \Gamma(\tfrac{1}{2}) = \sqrt{\pi} \quad ; \; n = 1, 2, 3, \dots$

ii) Rekurence : $\Gamma(z+1) = z\,\Gamma(z)$ (dodefinovává $\Gamma$ pro $\forall z \in \mathbb{C}$)

iii) Reflekční forule : $\Gamma(z)\Gamma(1-z) = \dfrac{\pi}{\sin \pi z} \quad ; \; \forall z \in \mathbb{C} / \mathbb{Z}$

iv) Duplicitní formule : $\Gamma(2z) = \dfrac{1}{\sqrt{\pi}}\, 2^{2z-1}\, \Gamma(z)\Gamma(z+\tfrac{1}{2})$

v) Weierstrassův součin : $\Gamma(z) = \dfrac{e^{-\gamma z}}{z} \prod_{n=1}^{\infty} \dfrac{e^{\frac{z}{n}}}{1 + \frac{z}{n}}$

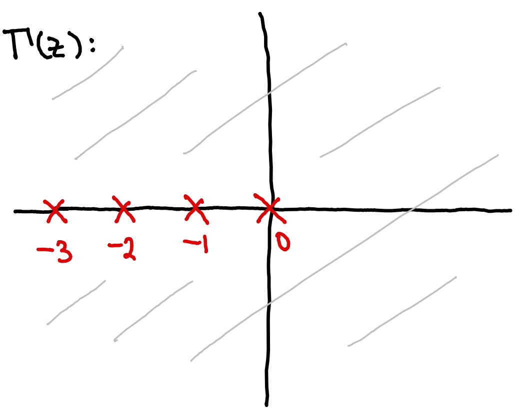
$\left[ \gamma := \lim_{n\to\infty} \left( \ln n - \underbrace{\sum_{k=1}^{n} \frac{1}{k}}_{H_n} \right) \right.$ Euler–Mascheroni $\Big]$

vi) Póly : $\lim_{z \to -n} (z+n)\,\Gamma(z) = \dfrac{(-1)^n}{n!} \quad ; \; n = 0, 1, 2, \dots$

vii) Asymptotika v $\infty$ :

$$\Gamma(z) \cong \sqrt{\frac{2\pi}{z}}\, z^z e^{-z}$$

$\Gamma(z)$:




# BETA FUNKCE

$\boxed{D}$ $B(u,v) := \int_0^1 t^{u-1} (1-t)^{v-1}\, dt$

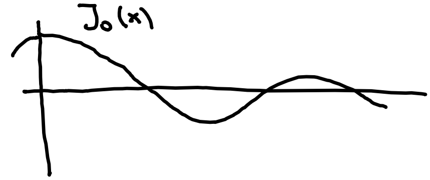
$\boxed{V}$ (VLASTNOSTI $B$ FUNKCE)

i) $B(u,v) = \dfrac{\Gamma(u)\,\Gamma(v)}{\Gamma(u+v)}$

ii) $\int_0^{\frac{\pi}{2}} \cos^{\ell}\theta \, \sin^{n}\theta \, d\theta = \frac{1}{2} B\left(\frac{\ell+1}{2}, \frac{n+1}{2}\right)$

# BESSELOVY FUNKCE

## I. DRUHU :




$\boxed{D}$ $$J_\nu(x) := \sum_{\ell=0}^{\infty} \frac{(-1)^\ell}{\ell!\,\Gamma(\ell+\nu+1)} \left(\frac{x}{2}\right)^{2\ell+\nu}$$

$\boxed{V}$ (VLASTNOSTI BESSELOVÝCH FUNKCÍ)

i) ODE $(\nu \in \mathbb{R})$ : $x^2 J_\nu'' + x J_\nu' + (x^2 - \nu^2) J_\nu = 0$

ii) Integrální reprezentace : $J_n(x) = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{in\theta - ix\sin\theta}\, d\theta$

[varianta : $J_n(x) = \frac{(-i)^n}{2\pi}\int_0^{2\pi} e^{in\theta + ix\cos\theta}\, d\theta$ ]

iii) Asymptotika $J_\nu(z) \overset{|z|\gg 1}{=} \sqrt{\frac{2}{\pi z}} \cos\left(z - \frac{\nu\pi}{2} - \frac{\pi}{4}\right) + O\left(\frac{1}{|z|^{3/2}}\right)$

iv) Speciální hodnoty : $J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\cos x$ ; $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\sin x$

[PŘESNÉ !]

## II. DRUHU :

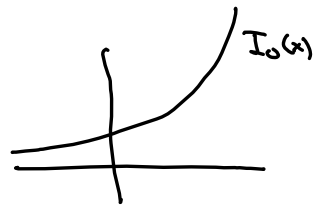
$\boxed{D}$ $$Y_\nu(z) := \frac{J_\nu(z)\cos(\pi\nu) - J_{-\nu}(z)}{\sin(\pi\nu)}$$ (STEJNÁ ODE ?)

i) Integrální reprezentace :

ii) Asymptotika : $Y_\nu(z) \overset{|z|\gg 1}{=} \sqrt{\frac{2}{\pi z}} \sin\left(z - \frac{\nu\pi}{2} - \frac{\pi}{4}\right) + O\left(\frac{1}{|z|^{3/2}}\right)$

# MODIFIKOVANÉ BESSELOVY FCE

## I. DRUHU:




$\boxed{D}$ $$I_\nu(x) := \sum_{l=0}^{\infty} \frac{1}{l!\,\Gamma(l+\nu+1)} \left(\frac{x}{2}\right)^{2l+\nu}$$

$\boxed{V}$ (VLASTNOSTI MOD. BESS. FUNKCÍ)

i) ODE ($\nu \in \mathbb{R}$): $x^2 I_\nu'' - x I_\nu' - (x^2+\nu^2) I_\nu =$

ii) Integrální reprezentace:

$$I_\nu(x) = \frac{1}{\pi}\int_0^\pi e^{x\cos\theta} \cos(\nu\theta)\, d\theta - \frac{\sin(\pi\nu)}{\pi}\int_0^\infty e^{-x\cosh t - \nu t}\, dt$$

iii) Speciální hodnoty: $I_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \sinh x$ $\left(\approx I_\nu(x);\ x \to \infty\right)$

## II. DRUHU:

$\boxed{D}$ $$K_\nu(z) := \frac{\pi}{2} \frac{I_{-\nu}(z) - I_\nu(z)}{\sin(\pi\nu)}$$ (STEJNÁ ODE 8)

i) Integrální reprezentace: $K_0(x) = \int_0^\infty \frac{\cos(tx)}{\sqrt{1+t^2}}\, dt$

[ obecně $K_\nu(x) = \int_0^\infty e^{-x\cosh t} \cosh(\nu t)\, dt$ ]

ii) Speciální hodnoty: $K_{\frac{1}{2}}(x) = \sqrt{\frac{\pi}{2x}}\, e^{-x}$ $\left(\approx K_\nu(x);\ x \to \infty\right)$