Life insurance II – a simple cash-flow model and a simulation of the net premium reserve

Computations using the commutations functions

Use the Unisex Life Tables and commutation functions (CF), and the technical interest rate 1.3%.

1. Consider a portfolio of endowments valid to age 65 of the insured person. You are given the sum insured SI, the underwriting year and the age x of the insured person when the contract started. We assume that all contracts started at January 1.

2. Using the commutation functions, estimate the net annual premium which is paid during the whole contract life.

3. For each contract compute the net premium reserve to January 1, 2018 using the commutation functions.

You should be always sure what is the contract length n and the date of reserve computation k. They are different for each contract.

Simulation part

Using a suitable software tool (Mathematica, Matlab, R etc.) simulate the future development of the contract and using these simulations estimate the net premium reserve to January 1, 2018.

1. Using the probabilities based on Life Tables you can simulate the death of the insured person and model the cash-flow related to the death or survival.

2. For discounting use a general interested rate (yield curve) which is simulated using the discretized CIR process

$$r_t = r_{t-1} + \alpha(\mu - r_{t-1}) + \sigma\sqrt{r_{t-1}}\epsilon_t, \tag{1}$$

   where $\alpha = 0.5$, $\mu = 0.013$, $\sigma = 0.01$ and $\epsilon_t \sim N[0,1]$ are i.i.d. random variables. Do not use the negative values[1], although maybe it makes sense today:)

3. For each simulation of the yield curve and for each contract compute the discounted cash-flow.

4. Using the sum over all contracts for each yield curve (you will get 1000 simulations for the whole portfolio), estimate the descriptive statistics for the net premium reserve: mean, standard deviation, minimum, maximum, lower and upper quartile, median. The mean should be "close" to the estimate of the NPR using CF.

Send a pdf file surname_name_HW12.pdf **to my e-mail until March 26, 2018 with**:

- the net annual premium and the net premium reserve computed using the commutation functions,

[1] Fix the scenario to the last positive value or start a new simulation, until you get 1000 positive scenarios.

- the descriptive statistics for the simulations of the net premium reserve,
- YOUR commented source code (as an appendix).