

Life insurance II – net premium reserve for a portfolio

HW 1: Computations using the commutations functions

Use the Unisex Life Tables and commutation functions (CF), and the technical interest rate 1.2%.

1. Consider a portfolio of life annuities due which are deferred to age 65 of the insured person. You are given the sum insured SI , the underwriting year and the age x of the insured person when the contract started. We assume that all contracts started at January 1.
2. Using the commutation functions, estimate the net annual premium which is paid during the whole contract life.
3. For each contract compute the net premium reserve to January 1, 2020 using the commutation functions.

You should be always sure what is the length of deferment period m and the date of reserve computation k . They are different for each contract. Some contracts can be even after deferment.

HW 2: Simulation

Using a suitable software tool (Mathematica, Matlab, R etc.¹) simulate the future development of the contract and using these simulations estimate the net premium reserve to January 1, 2020.

1. Using the probabilities based on Life Tables you can simulate the death of the insured person (p_x is the survival probability of a person at age x) and model the cash-flow related to the death or survival.
2. For discounting use a general interested rate (yield curve) which is simulated using the discretized CIR process

$$r_t = r_{t-1} + \alpha(\mu - r_{t-1}) + \sigma\sqrt{r_{t-1}}\epsilon_t, \quad (1)$$

where $r_0 = 0.012$, $\alpha = 0.5$, $\mu = 0.012$, $\sigma = 0.01$ and $\epsilon_t \sim N[0, 1]$ are i.i.d. random variables. Do not use the negative values². For discounting use

$$\frac{1}{1+r_1}, \frac{1}{(1+r_1)(1+r_2)}, \dots \quad (2)$$

3. For each simulation of the yield curve and for each contract simulate the year of death and compute the discounted cash-flows.
4. Using the sum over all contracts for each yield curve, you will get 1000 simulations for the whole portfolio. Use these aggregated simulations to estimate the descriptive statistics for the net premium reserve: mean, standard deviation, minimum, maximum, lower and upper quartile, median. The mean should be "close" to the estimate of the NPR using CF.

¹MS Excel is not acceptable, although I know that VBA is quite powerful.

²Fix the scenario to the last positive value or start a new simulation, until you get 1000 positive scenarios.

Send a pdf file surname_name_HW12.pdf to my e-mail until April 1, 2020 with:

- the net annual premium and the net premium reserve computed using the commutation functions,
- the descriptive statistics for the simulations of the net premium reserve,
- YOUR commented source code (as an appendix).

Please deliver you HW in a readable format, e.g. 2 page summary + appendix³.

³Title page with matfyz logo is not necessary :), BUT "long code - hidden results - long code - several numbers" is not the acceptable format.