

# NMFM405 Life Insurance – Exercises

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## 1 Financial mathematics

**Example 1.1.** Mrs Y will need 150 000 CZK after 5 years to pay for a transfer of the "flat rights". The bank offers a term account with nominal interest rate 4.4% credited quarterly. How much money must Mrs Y save to cover the necessary amount for the transfer?

**Example 1.2.** Mr Z bought a house for 3 600 000 Kč using mortgage loan of amount 2 500 000 CZK. He will repay 16 000 CZK monthly in arrear over 25 years. What is the nominal annual interest rate which the bank offered to Mr Z?

**Example 1.3.** Mrs Y makes deposits of 100 at time 0, and  $x$  at time 3. The fund grows at a force of interest

$$\delta_t = \frac{t^2}{100}, t > 0.$$

Let the amount of interest earned from time 3 to 6 is also equal to  $x$ . Calculate  $x$ .

**Example 1.4.** Mr X wants to borrow 150 000\$. He would like to repay this loan in 2 years by periodic semiannual payments. Bank offers the nominal interest rate 6.9%. Mr X has 45 000\$ on his account where the interest is credited monthly under nominal interest rate 2.5%. He can save 40 000\$ every half a year from his salary.

0. Plot cash-flow.

1. Will Mr X have enough money on his account to cover the semiannual loan payments?

2. How much money should he hold at the beginning to cover the loan payments?

Solution: 2. Loan repayments  $R$ , savings  $S = 40000$ , nominal interest rate  $j_{(12)} = 0.025$ .

Cash-flow balance equation: future value after 2 years should be zero ...

$$FV_{2y} = 0 = \left\{ \left[ \left( \text{Init. money} \left( 1 + \frac{j_{(12)}}{12} \right)^6 - R + S \right) \left( 1 + \frac{j_{(12)}}{12} \right)^6 - R + S \right] \dots \right. \\ \left. \dots \left( 1 + \frac{j_{(12)}}{12} \right)^6 - R + S \right\} \left( 1 + \frac{j_{(12)}}{12} \right)^6 - R + S.$$

This leads to

$$\text{Initial money} = \sum_{i=1}^4 \frac{(R - S)}{\left( 1 + \frac{j_{(12)}}{12} \right)^{6i}}$$

**Example 1.5.** Mr X sold his car for 200 000 CZK. He paid this amount to his account where the interest is credited monthly with nominal interest rate 2.8%. He decided to buy a new car 9 months after. There was a necessary advance payment of 50 000 CZK taken from the account. Then, the debt was repaid by payments 9 000 CZK monthly. How long the money on the account can cover these payments?

Solution:

$$200000 \left(1 + \frac{0.028}{12}\right)^9 - 50000 = 9000a_n^{(12)}.$$

Express  $n$ .

**Example 1.6.** Calculate the net present value for a bond with the nominal value 1000 \$, annual coupon rate 6% and term to maturity 3 years. Consider a yield curve with annual spot/forward interest rates 3, 4, 5 %.

Solution:

$$PV = \frac{60}{1 + 0.03} + \frac{60}{(1 + 0.03)(1 + 0.04)} + \frac{1000 + 60}{(1 + 0.03)(1 + 0.04)(1 + 0.05)}.$$