Homework 3 – Nonlinear programming in portfolio optimization

Martin Branda

Charles University in Prague
Faculty of Mathematics and Physics
Department of Probability and Mathematical Statistics

Computational Aspects of Optimization

Mean–variance–skewness portfolio optimization

Consider n assets with random rates of return denoted by R_i, with $\mathbb{E}|R_i|^3 < \infty$ and define the corresponding covariance matrix C and skewness tensor S elementwise as

$$C_{jk} := \mathbb{E}(R_j - \mathbb{E}R_j)(R_k - \mathbb{E}R_k),$$
$$S_{jkl} := \mathbb{E}(R_j - \mathbb{E}R_j)(R_k - \mathbb{E}R_k)(R_l - \mathbb{E}R_l).$$

Employ the aggregate function approach of multiobjective optimization with aggregation parameter $c > 0$

$$\text{minimize} \sum_{j=1}^{n}\sum_{k=1}^{n} C_{jk}x_j x_k - c\sum_{j=1}^{n}\sum_{k=1}^{n}\sum_{l=1}^{n} S_{jkl}x_j x_k x_l$$
$$\sum_{i=1}^{n} \mathbb{E}[R_i]\cdot x_i \geq r_0, \tag{1}$$
$$\sum_{i=1}^{n} x_i = 1,\ x_i \geq 0.$$

Homework 3

1. Use the same data as for the CVaR and VaR homework to estimate the mean vector, variance matrix and skewness tensor.
2. Solve the mean–variance ($c = 0$) and the mean–variance–skewness[1] ($c = 0.1$) problems.
3. Solve the problems for different 11 values $r_0 \in \{\min_i \overline{R}_i, \ldots, \max_i \overline{R}_i\}$.
4. Plot the optimal values against the corresponding values of r_0.

Use both Matlab and GAMS.

[1] The problem is nonconvex in general.