

A. Vyšetřete konvergenci integrálů:

$$1. \int_0^{\infty} \frac{\sqrt[3]{x} + 3}{\sqrt{x} + 2} dx \quad 2. \int_0^1 \frac{\sin(\pi x)}{x^{9/8} \ln x} dx \quad 3. \int_0^{\infty} \frac{\ln(x+1)}{x^{101/100}} dx$$

$$4. \int_0^{\infty} \frac{\sin^2 x}{x^a} dx \quad 5. \int_0^{\infty} \sin^3\left(\frac{1}{\sqrt{x}}\right) dx \quad 6. \int_0^{\pi} \frac{dx}{\sqrt{\sin x}}$$

$$7. \int_0^{\infty} x \exp(-x) \sin x dx \quad * 8. \int_0^{\infty} \frac{x}{1 + x^5 \sin^2 x} dx \quad 9. \int_0^{1/2} \frac{dx}{x \ln^b x}$$

$$10. \int_0^{\infty} \frac{dx}{\exp x - \cos x} \quad 11. \int_0^{\infty} \frac{\operatorname{arctg} x}{x^a} dx \quad * 12. \int_0^{\infty} \frac{p(x)}{q(x)} dx$$

B. Záměnou limity a integrálu vypočtete:

$$1. \lim_{n \rightarrow \infty} \int_0^{\infty} \exp(-x^n) dx \quad 2. \lim_{n \rightarrow \infty} \int_0^{10} \frac{nx}{1 + n^2 x^2} dx$$

$$3. \lim_{n \rightarrow \infty} \int_0^{\infty} \frac{dx}{1 + x + x^2 n^{-2}} \quad 4. \lim_{n \rightarrow \infty} \int_0^{\infty} \frac{\operatorname{arctg}(nx)}{1 + x^3} dx$$

$$5. \lim_{n \rightarrow \infty} \int_1^{\infty} \frac{dx}{\ln x + \ln n} \quad 6. \lim_{n \rightarrow \infty} \int_0^{\infty} \frac{x^n}{1 + x^{2n}} dx$$

$$7. \lim_{n \rightarrow \infty} \int_0^{\infty} \frac{\ln(x+n)}{n} \exp(-x) \cos x dx \quad 8. \lim_{n \rightarrow \infty} \int_0^{23} \frac{\exp(x^3)}{1 + n\sqrt{x}} dx$$

C. Rozviňte v řadu:

$$1. \int_0^1 \ln\left(\frac{1+x}{1-x}\right) dx \quad 2. \int_0^{\infty} \exp(-x) \cos \sqrt{x} dx \quad 3. \int_0^1 \ln(x) \ln(1-x) dx$$

$$4. \int_0^1 \frac{x^p \ln x}{1+x^2} dx \quad 5. \int_0^{\infty} \frac{\sin x}{1 + \exp x} dx \quad 6. \int_0^{\infty} \frac{x}{\exp x - 1} dx$$