

## EXTRÉMY FUNKCÍ VÍCE PROMĚNNÝCH II

Nalezněte extrémy funkce  $f$  na množině  $M$ .

*V těchto úlohách použijte Lagrangeovu větu o multiplikatorech.*

- $f(x, y, z) = xyz,$   
 $M = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 = 1\}$
- $f(x, y, z) = xyz,$   
 $M = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 = 1, x + y + z = 0\}$
- $f(x, y) = xyz,$   
 $M = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 + z^2 \leq 1, x + y + z \leq 0\}$
- $f(x, y, z) = \sin x \sin y \sin z$   
 $M = \left\{ [x, y, z] \in \mathbb{R}^3 : x + y + z = \frac{\pi}{2}, x > 0, y > 0, z > 0 \right\}$
- $f(x, y, z) = xy^2z^3$   
 $M = \{ [x, y, z] \in \mathbb{R}^3 : x + 2y + 3z = a, x > 0, y > 0, z > 0 \},$   
kde  $a > 0$
- $f(x, y) = y$   
 $M = \{ [x, y] \in \mathbb{R}^2 : (x^2 + y^2)^2 - 2(x^2 - y^2) = 0 \}$
- $f(x, y) = x + y,$   
 $M = \{ [x, y] \in \mathbb{R}^2 : x^3 + y^3 - 2xy = 0, x \geq 0, y \geq 0 \}$

## VÝSLEDKY

1. maximum v  $[1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3}]$ ,  $[1/\sqrt{3}, -1/\sqrt{3}, -1/\sqrt{3}]$ ,  $[-1/\sqrt{3}, 1/\sqrt{3}, -1/\sqrt{3}]$ ,  
 minimum v  $[-1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3}]$ ,  $[1/\sqrt{3}, -1/\sqrt{3}, 1/\sqrt{3}]$ ,  $[1/\sqrt{3}, 1/\sqrt{3}, -1/\sqrt{3}]$ ,  
 $[-1/\sqrt{3}, -1/\sqrt{3}, -1/\sqrt{3}]$
2. maximum v  $\left[\frac{2}{\sqrt{6}}, -\frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}}\right]$ ,  $\left[-\frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, -\frac{1}{\sqrt{6}}\right]$ ,  $\left[-\frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}\right]$   
 minimum v  $\left[-\frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right]$ ,  $\left[\frac{1}{\sqrt{6}}, -\frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right]$ ,  $\left[\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, -\frac{2}{\sqrt{6}}\right]$
3. maximum v  $[\pi/6, \pi/6, \pi/6]$ ,  
 minima se nenabývá
4. maximum v  $[a/6, a/6, a/6]$ ;  
 minima se nenabývá.
5. maximum v  $[\sqrt{3}/2, 1/2]$ ,  $[-\sqrt{3}/2, 1/2]$ ;  
 minimum v  $[\sqrt{3}/2, -1/2]$ ,  $[-\sqrt{3}/2, -1/2]$ .
6. maximum v  $\left[\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right]$   
 minimum v  $\left[-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right]$ ,  $\left[\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right]$ ,  $\left[\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\right]$
7. minimum v  $[0, 0]$ ,  
 maximum v  $[1, 1]$