

IMPLICITNĚ ZADANÉ FUNKCE

Ukažte, že uvedená rovnice určuje na nějakém okolí daného bodu $[\tilde{x}, \tilde{y}]$ implicitně zadanou funkci¹ φ proměnné x . Spočítejte $\varphi'(\tilde{x})$, $\varphi''(\tilde{x})$.

1. $x^2 + 2xy^2 + y^4 - y^5 = 0$, $[\tilde{x}, \tilde{y}] = [0, 1]$
2. $e^{xy} + \sin y + y^2 = 1$, $[\tilde{x}, \tilde{y}] = [2, 0]$
3. $\sin(xy) + \cos(xy) = 1$, $[\tilde{x}, \tilde{y}] = [\pi, 0]$
4. $2x^4y + x^3 + y^3 + xy = 1$, $[\tilde{x}, \tilde{y}] = [1, 0]$
5. $\log(x^2 + y^2 + \cos(xy)) + y = 0$, $[\tilde{x}, \tilde{y}] = [0, 0]$
6. $\log(x + \arctg y + 1) + xy = 0$, $[\tilde{x}, \tilde{y}] = [0, 0]$
7. $x^y + y^x = 2y$, $[\tilde{x}, \tilde{y}] = [1, 1]$
8. $y^3x^2 + y^2x^2 + \sin y = 0$, $[\tilde{x}, \tilde{y}] = [0, 0]$
9. $e^{\sin x^2} + e^{\sin xy} = 2y + 2$, $[\tilde{x}, \tilde{y}] = [0, 0]$
10. $\frac{\pi}{2} + \arcsin(x + y^2) = \arccos(y + x^2)$, $[\tilde{x}, \tilde{y}] = [0, 0]$

Ukažte, že uvedená rovnice určuje na nějakém okolí daného bodu $[\tilde{x}, \tilde{y}, \tilde{z}]$ implicitně zadanou funkci z proměnných x, y . Spočítejte $\frac{\partial z}{\partial x}(\tilde{x}, \tilde{y})$, $\frac{\partial z}{\partial y}(\tilde{x}, \tilde{y})$.

11. $x^2 + 2y^2 + 3z^2 + xy - z - 9 = 0$, $[\tilde{x}, \tilde{y}, \tilde{z}] = [1, -2, 1]$
12. $x^2 + y^2 + z^2 - 3xyz = 0$, $[\tilde{x}, \tilde{y}, \tilde{z}] = [1, 1, 1]$
13. $\frac{x}{z} = \log \frac{z}{y}$, $[\tilde{x}, \tilde{y}, \tilde{z}] = [0, 1, 1]$

Ukažte, že uvedená soustava rovnic určuje na nějakém okolí daného bodu $[\tilde{x}, \tilde{y}, \tilde{u}, \tilde{v}]$ implicitně zadané funkce u, v proměnných x, y . Spočítejte $\frac{\partial u}{\partial x}(\tilde{x}, \tilde{y})$, $\frac{\partial v}{\partial x}(\tilde{x}, \tilde{y})$, $\frac{\partial u}{\partial y}(\tilde{x}, \tilde{y})$, $\frac{\partial v}{\partial y}(\tilde{x}, \tilde{y})$.

14. $xe^{u+v} + 2uv = 1, ye^{u-v} - \frac{u}{1+v} = 2x$, $[\tilde{x}, \tilde{y}, \tilde{u}, \tilde{v}] = [1, 2, 0, 0]$
15. $x = u \cos \frac{v}{u}, y = u \sin \frac{v}{u}$, $[\tilde{x}, \tilde{y}, \tilde{u}, \tilde{v}] = [1, 0, 1, 0]$
16. $x = e^u + u \sin v, y = e^u - u \cos v$, $[\tilde{x}, \tilde{y}, \tilde{u}, \tilde{v}] = [e + 1, e, 1, \pi/2]$

¹tzn. v tomto okolí je splněna rovnice, právě když $y = \varphi(x)$

Ukažte, že uvedená soustava rovnic určuje na nějakém okolí daného bodu $[\tilde{x}, \tilde{y}, \tilde{z}, \tilde{u}, \tilde{v}, \tilde{w}]$ implicitně zadané funkce x, y, z proměnných u, v, w . Spočítejte zadaný gradient.²

17.
$$\begin{cases} u = \arctg(\pi x) + y^2 z, \\ v = e^{-x} + 2\frac{y}{z}, \\ w = \cos(2xy) + 2\sqrt{z} \end{cases} \quad \begin{aligned} [\tilde{u}, \tilde{v}, \tilde{w}] &= [4, \frac{3}{2}, 5], \\ [\tilde{x}, \tilde{y}, \tilde{z}] &= [0, 1, 4] \\ \nabla y(\tilde{u}, \tilde{v}, \tilde{w}) &= ? \end{aligned}$$
18.
$$\begin{cases} u = \sin x + xy + e^z, \\ v = \cos y + xe^{-y}, \\ w = x^2 + 2y - \cos(xz) \end{cases} \quad \begin{aligned} [\tilde{u}, \tilde{v}, \tilde{w}] &= [1 + \sin 1, 2, 0], \\ [\tilde{x}, \tilde{y}, \tilde{z}] &= [1, 0, 0] \\ \nabla x(\tilde{u}, \tilde{v}, \tilde{w}) &= ? \end{aligned}$$
19.
$$\begin{cases} u = \sin(\pi xy) - \arctg\left(\frac{y}{x}\right), \\ v = 3xy^2 + e^{x+y} \end{cases} \quad \begin{aligned} [\tilde{u}, \tilde{v}] &= [\frac{\pi}{4}, -2], \\ [\tilde{x}, \tilde{y}] &= [-1, 1] \\ \frac{\partial z}{\partial u}(\tilde{u}, \tilde{v}) &= ?, \text{ kde } z(x, y) = x^2 + y^2. \end{aligned}$$
20.
$$\begin{cases} u = \text{tg}(xy) + \log(x^2 + y^2), \\ v = \arcsin(2x) - (3^x)^y \end{cases} \quad \begin{aligned} [\tilde{u}, \tilde{v}] &= [0, -1], \\ [\tilde{x}, \tilde{y}] &= [0, 1] \\ \nabla y(\tilde{u}, \tilde{v}) &= ? \end{aligned}$$
21.
$$\begin{cases} u = \cos(\pi y) + 2\sqrt{xz}, \\ v = \frac{y}{x} - \log x, \\ w = \frac{x^2}{2} - \arctg(yz) \end{cases} \quad \begin{aligned} [\tilde{u}, \tilde{v}, \tilde{w}] &= [3, 0, \frac{1}{2}], \\ [\tilde{x}, \tilde{y}, \tilde{z}] &= [1, 0, 1] \\ \nabla y(\tilde{u}, \tilde{v}, \tilde{w}) &= ? \end{aligned}$$

²Uvědomte si, že z Věty o implicitních funkcích jsou příslušné funkce \mathcal{C}^1 , a proto existuje totální diferenciál, určený příslušným gradientem.

VÝSLEDKY

1. $\varphi'(\tilde{x}) = 2, \varphi''(\tilde{x}) = -14$
2. $\varphi'(\tilde{x}) = 0, \varphi''(\tilde{x}) = 0$
3. $\varphi'(\tilde{x}) = 0, \varphi''(\tilde{x}) = 0$
4. $\varphi'(\tilde{x}) = -1, \varphi''(\tilde{x}) = 4$
5. $\varphi'(\tilde{x}) = 0, \varphi''(\tilde{x}) = -2$
6. $\varphi'(\tilde{x}) = -1, \varphi''(\tilde{x}) = 2$
7. $\varphi'(\tilde{x}) = 1, \varphi''(\tilde{x}) = 4$
8. $\varphi'(\tilde{x}) = 0, \varphi''(\tilde{x}) = 0$
9. $\varphi'(\tilde{x}) = 0, \varphi''(\tilde{x}) = 1$
10. $\varphi'(\tilde{x}) = -1, \varphi''(\tilde{x}) = -4$
11. $\frac{\partial z}{\partial x}(\tilde{x}, \tilde{y}) = 0, \frac{\partial z}{\partial y}(\tilde{x}, \tilde{y}) = \frac{7}{5}$
12. $\frac{\partial z}{\partial x}(\tilde{x}, \tilde{y}) = -1, \frac{\partial z}{\partial y}(\tilde{x}, \tilde{y}) = -1$
13. $\frac{\partial z}{\partial x}(\tilde{x}, \tilde{y}) = 1, \frac{\partial z}{\partial y}(\tilde{x}, \tilde{y}) = 1$
14. $\frac{\partial u}{\partial x}(\tilde{x}, \tilde{y}) = 0, \frac{\partial v}{\partial x}(\tilde{x}, \tilde{y}) = -1,$
 $\frac{\partial u}{\partial y}(\tilde{x}, \tilde{y}) = -1/3, \frac{\partial v}{\partial y}(\tilde{x}, \tilde{y}) = 1/3$
15. $\frac{\partial u}{\partial x}(\tilde{x}, \tilde{y}) = 1, \frac{\partial v}{\partial x}(\tilde{x}, \tilde{y}) = 0,$
 $\frac{\partial u}{\partial y}(\tilde{x}, \tilde{y}) = 0, \frac{\partial v}{\partial y}(\tilde{x}, \tilde{y}) = 1$
16. $\frac{\partial u}{\partial x}(\tilde{x}, \tilde{y}) = 1/(1+e), \frac{\partial v}{\partial x}(\tilde{x}, \tilde{y}) = -e/(1+e),$
 $\frac{\partial u}{\partial y}(\tilde{x}, \tilde{y}) = 0, \frac{\partial v}{\partial y}(\tilde{x}, \tilde{y}) = 1$
17. $\nabla y(\tilde{u}, \tilde{v}, \tilde{w}) = \left(\frac{2}{\pi+16}, \frac{2\pi}{\pi+16}, \frac{\pi-8}{\pi+16} \right),$
18. $\nabla x(\tilde{u}, \tilde{v}, \tilde{w}) = \left(0, \frac{1}{2}, \frac{1}{4} \right),$
19. $\frac{\partial z}{\partial u}(\tilde{u}, \tilde{v}) = -\frac{2}{9-2\pi}$
20. $\nabla y(\tilde{u}, \tilde{v}) = \left(\frac{1}{2-\log 3}, \frac{-1}{2(2-\log 3)} \right)$
21. $\nabla x(\tilde{u}, \tilde{v}, \tilde{w}) = \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2} \right),$