PARCIÁLNÍ DERIVACE

Spočtěte parciální derivace funkcí všude, kde existují.

1. $f(x,y) = x^m y^n$, kde $m, n \in \mathbb{N}$

2. $f(x,y) = e^{xy}$

3. $f(x,y,z) = xy + yz + zx$

4. $f(x,y) = |x| \cdot |y|$

5. $f(x,y,z) = \left(\dfrac{x}{y}\right)^z$

6. $f(x,y,z) = x^{\frac{y}{z}}$

7. $f(x,y) = \sqrt{x + y^2}$

8. $f(x,y) = \sqrt{x^2 + y^2}$

9. $f(x,y) = \sqrt[3]{x^3 + y^3}$

10. $f(x,y) = \sqrt[3]{xy}$

11. $f(x,y) = |y - \sin x|$

12. $f(x,y) = |\sin y - \sin x|$

13. $f(x,y) = e^{\frac{-1}{x^2+xy+y^2}}$, $f(0,0) = 0$

14. $f(x,y,z) = x^{y^z}$

VÝSLEDKY

1. $\frac{\partial f}{\partial x} = mx^{m-1}y^n$, $\frac{\partial f}{\partial y} = nx^m y^{n-1}$ pro $[x,y] \in \mathbb{R}^2$.

2. $\frac{\partial f}{\partial x} = ye^{xy}$, $\frac{\partial f}{\partial y} = xe^{xy}$ pro $[x,y] \in \mathbb{R}^2$.

3. $\frac{\partial f}{\partial x} = y + z$, $\frac{\partial f}{\partial y} = x + z$, $\frac{\partial f}{\partial z} = x + y$ pro $[x,y,z] \in \mathbb{R}^3$.

4. $\frac{\partial f}{\partial x}(x,y) = |y| \operatorname{sgn} x$ pro $x \neq 0$. $\frac{\partial f}{\partial y}(x,y) = |x| \operatorname{sgn} y$ pro $y \neq 0$. $\frac{\partial f}{\partial x}(0,0) = \frac{\partial f}{\partial y}(0,0) = 0$. $\frac{\partial f}{\partial x}(0,y)$ pro $y \neq 0$ a $\frac{\partial f}{\partial y}(x,0)$ pro $x \neq 0$ neexistují.

5. Pokud $x, y > 0$ nebo $x, y < 0$, pak $\frac{\partial f}{\partial x} = \frac{z}{y} \cdot \left(\frac{x}{y}\right)^{z-1}$; $\frac{\partial f}{\partial y} = -\frac{zx}{y^2} \cdot \left(\frac{x}{y}\right)^{z-1}$; $\frac{\partial f}{\partial z} = \left(\frac{x}{y}\right)^{z} \cdot \log \frac{x}{y}$.

6. Pokud $x > 0$ a $z \neq 0$, pak $\frac{\partial f}{\partial x} = \frac{y}{z} \cdot x^{\frac{y}{z}-1}$; $\frac{\partial f}{\partial y} = x^{\frac{y}{z}} \cdot \log x \cdot \frac{1}{z}$; $\frac{\partial f}{\partial z} = -x^{\frac{y}{z}} \cdot \log x \cdot \frac{y}{z^2}$.

7. Pokud $x > -y^2$, pak $\frac{\partial f}{\partial x} = \frac{1}{2\sqrt{x+y^2}}$; $\frac{\partial f}{\partial y} = \frac{y}{\sqrt{x+y^2}}$.

8. $\frac{\partial f}{\partial x}(x,y) = \dfrac{x}{\sqrt{x^2+y^2}}$, $\frac{\partial f}{\partial y}(x,y) = \dfrac{y}{\sqrt{x^2+y^2}}$, pokud $[x,y] \neq [0,0]$. $\frac{\partial f}{\partial x}(0,0)$ a $\frac{\partial f}{\partial y}(0,0)$ neexistují.

9. $\frac{\partial f}{\partial x}(x,y) = \dfrac{x^2}{\sqrt[3]{(x^3+y^3)^2}}$, $\frac{\partial f}{\partial y}(x,y) = \dfrac{y^2}{\sqrt[3]{(x^3+y^3)^2}}$, pokud $y \neq -x$. $\frac{\partial f}{\partial x}(0,0) = \frac{\partial f}{\partial y}(0,0) = 1$, $\frac{\partial f}{\partial x}(x,-x)$ a $\frac{\partial f}{\partial y}(x,-x)$ neexistují pro $x \neq 0$.

10. $\frac{\partial f}{\partial x}(x,y) = \dfrac{\sqrt[3]{y}}{3\sqrt[3]{x^2}}$ pro $x \neq 0$. $\frac{\partial f}{\partial y}(x,y) = \dfrac{\sqrt[3]{x}}{3\sqrt[3]{y^2}}$ pro $y \neq 0$. $\frac{\partial f}{\partial x}(0,0) = \frac{\partial f}{\partial y}(0,0) = 0$. $\frac{\partial f}{\partial x}(0,y)$ pro $y \neq 0$ a $\frac{\partial f}{\partial y}(x,0)$ pro $x \neq 0$ neexistují.

11. $\frac{\partial f}{\partial x} f(x,y) = -\operatorname{sgn}(y - \sin x) \cdot \cos x$, $\frac{\partial f}{\partial y}(x,y) = \operatorname{sgn}(y - \sin x)$, pokud $y \neq \sin x$. $\frac{\partial f}{\partial y}(x, \sin x)$ neexistuje pro $x \in \mathbb{R}$. $\frac{\partial f}{\partial x} f(\dfrac{\pi}{2} + k\pi, (-1)^k) = 0$ pro $k \in \mathbb{Z}$. $\frac{\partial f}{\partial x} f(x, \sin x)$ neexistuje pro $x \neq \dfrac{\pi}{2} + k\pi$.

12. $\frac{\partial f}{\partial x}(x,y) = \cos x \operatorname{sgn}(\sin x - \sin y)$, $\frac{\partial f}{\partial y}(x,y) = -\cos y \operatorname{sgn}(\sin x - \sin y)$, pokud $\sin x \neq \sin y$. $\frac{\partial f}{\partial x}(\dfrac{\pi}{2}+k\pi, \dfrac{\pi}{2}+l\pi) = \frac{\partial f}{\partial y}(\dfrac{\pi}{2}+k\pi, \dfrac{\pi}{2}+l\pi) = 0$. V ostatních bodech parciální derivace neexistují.

13. $\frac{\partial f}{\partial x} = e^{\frac{-1}{x^2+xy+y^2}} \cdot \dfrac{2x+y}{(x^2+xy+y^2)^2}$, $\frac{\partial f}{\partial y} = e^{\frac{-1}{x^2+xy+y^2}} \cdot \dfrac{x+2y}{(x^2+xy+y^2)^2}$ pro $[x,y] \neq [0,0]$; $\frac{\partial f}{\partial x} f(0,0) = 0$, $\frac{\partial f}{\partial y}(0,0) = 0$

14. Pokud $x, y > 0$, pak $\frac{\partial f}{\partial x} = y^z \cdot x^{y^z - 1}$; $\frac{\partial f}{\partial y} = x^{y^z} \cdot \log x \cdot z y^{z-1}$; $\frac{\partial f}{\partial z} = x^{y^z} \cdot \log x \cdot y^z \cdot \log y$.