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1. INTRODUCTION

Financial and insurance calculations become more and more frequent and helpful for many users not only in their profession life but sometimes even in their personal life. Therefore a survey of formulas of financial and insurance mathematics that can be applied to such calculations seems to be a suitable aid. In some cases one should use instead of the term *formula* more suitable terms of the type *method*, *procedure* or *algorithm* since the corresponding calculations cannot be simply summed up to a single expression, and a verbal description without introducing complicated symbols is more appropriate.

The survey has the following ambitions:

- The formulas should be applicable in practice: it has motivated their choice for this survey first and foremost. On the other hand it is obvious that by time one puts to use in practice seemingly very abstract formulas of higher mathematics, e.g. when pricing financial derivatives, evaluating financial risks, applying accounting principles based on fair values, choosing alternative risk transfers ARL in insurance, and the like.
- The formulas should be error-free (though such a goal is not achievable in full) since in the financial and insurance framework one publishes sometimes in a hectic way various untried formulas and methods that may be incorrect. Of course, the formulas are introduced here without proofs because their derivation is not the task of this survey.
- The formulas should be systematically sorted and described including a simple denotation that enables a quick and operative searching. Explanation and references to related parts of the survey are often attached to some formulas so that one can browse and look up in the text in an effective way. The detailed *Index* is also helpful for this purpose.
- The formulas should be presented in the form that is in average the most frequent and the most conventional one in practice.
- The formulas should be sufficiently self-contained. Therefore formulas of related disciplines (e.g. from statistics, theory of probability, demography and others) are also given in final chapters.

The mathematical level of the formulas and methods ranges from simple ones exploiting only an arithmetic to very sophisticated matters of higher mathematics (e.g. the stochastic calculus, and the like). The author hopes that users find in this survey their level of acceptability corresponding to the problems they solve. The survey contains also *Mathematical compendium* to remind some basic mathematical principles, and chapters that are related in direct or indirect way to financial and insurance analysis: *Descriptive and mathematical statistics*, *Econometrics*, *Index theory*, *Stochastic processes* and *Statistical analysis of time series*. One attaches also *List of symbols* for symbols that are frequent in the text (however, special symbols may be explained in the context of particular formulas).

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