Nonlinear Differential Equations

Practical 6: Pseudomonotone Operators

1. Let $A : X \to X'$ be a nonlinear operator on a real Banach space X. Show that
   (a) A monotone and hemicontinuous $\implies$ A satisfies the condition (M),
   (b) A uniformly continuous $\implies$ A satisfies the condition $(\mathrm{S})_+$.
2. Let $A, B : X \to X'$ be nonlinear operators on a real Banach space X. Show that
   (a) A satisfies $(\mathrm{S})_+$ and B strongly continuous $\implies$ $A + B$ satisfies $(\mathrm{S})_+$,
   (b) A satisfies (S) and B strongly continuous $\implies$ $A + B$ satisfies (S),
   (c) A satisfies (M) and B strongly continuous $\implies$ $A + B$ satisfies (M).
3. Let $A, B : X \to X'$ be nonlinear operators on a real Banach space X. Show that
   (a) A monotone and hemicontinuous $\implies$ A pseudomonotone,
   (b) A strongly continuous $\implies$ A pseudomonotone,
   (c) A demicontinuous and satisfies $(\mathrm{S})_+$ $\implies$ A pseudomonotone,
   (d) A pseudomonotone and B pseudomonotone $\implies$ $A + B$ pseudomonotone,
   (e) A pseudomonotone $\implies$ A satisfies (P),
   (f) A pseudomonotone $\implies$ A satisfies (M).