27.11.2023 — Homework 3

Finite Element Methods 1

Due date: 11th December 2023

Submit a PDF/scan of the answers to the following questions before the deadline via the *Study Group Roster* (*Záznamník učitele*) in SIS, or hand-in directly at the practical class on the 11th December 2023.

1. (1 point) Consider a triangulation $\mathcal{T}_h$ of $\Omega \subset \mathbb{R}^2$ consisting of simplices T with diameter h_T, and define by ϱ_T the diameter of the largest inscribed ball in T. Show that the condition
$$\frac{h_T}{\varrho_T} \leq \sigma, \qquad \text{for all } T \in \mathcal{T}_h, \tag{1.1}$$
where the constant σ is independent of T, is equivalent to the condition that all angles in all $T \in \mathcal{T}_h$ are bounded from below by a positive constant θ_0 independent of T.

2. Consider a triangulation $\mathcal{T}_h$ of $\Omega \subset \mathbb{R}^n$ consisting of simplices T with diameter h_T, define by ϱ_T the diameter of the largest inscribed ball in T, and assume that (1.1) holds.
   (a) (1 point) Show that $|T|$, for any $T \in \mathcal{T}_h$, satisfies the condition
   $$C_1 h_T^n \leq |T| \leq C_2 h_T^n,$$
   where C_1 is a positive constant dependent only on σ and n, and C_2 is a positive constant dependent only on n.
   (b) (1 point) Show, for $n = 3$, that any face F of $\mathcal{T}_h$ satisfies the condition.
   $$\frac{h_F}{\varrho_F} \leq \sigma.$$
   (c) (1 point) Show that $h_T \leq \sigma h_{\widetilde{T}}$, for $n = 2, 3$, for any pair of elements $T, \widetilde{T} \in \mathcal{T}_h$ sharing an edge.

3. (2 points) Let $\mathcal{T}_h$ be a triangulation consisting of n-simplices T in $\mathbb{R}^n$ satisfying (1.1) and the assumptions ($\mathcal{T}_h$1)–($\mathcal{T}_h$5). Prove that the number of elements of $\mathcal{T}_h$ sharing a vertex is bounded by a constant depending on σ and n.