# Numerical Solution of ODEs

Exercise Class

3rd October 2023

## Exercises

1. Read about the following ODE systems:

   **Population Growth** `http://en.wikipedia.org/wiki/Population_growth`
   **Logistic** `http://en.wikipedia.org/wiki/Logistic_function`
   **Pendulum** `http://en.wikipedia.org/wiki/Pendulum_%28mathematics%29`
   **Harmonic Oscillator** `http://en.wikipedia.org/wiki/Harmonic_oscillator`

2. Experiment with the logistic equation with the following constants, time range, and initial conditions $x_0$:

   - $a = b = 1$
   - $t = [0, 3], [0, -1]$
   - $x_0 = 1/2, 3/2, 1, -1/20$

3. Compare `ode23` and `ode15s` for the logistic equation with $a = b = 1$, c=1/5, $t = [0, 100]$, $x_0 = 0.7233$. Compare also to the exact solution:

$$u(t) = \frac{\sqrt{5}}{10} \tanh\left((t - t_0)\frac{\sqrt{5}}{10} + \operatorname{arctanh}\left((2x_0 - 1)\sqrt{5}\right)\right) + \frac{1}{2}.$$

4. Use `ode23` to solve the pendulum problem:

$$\begin{aligned} x''(t) &= -k \sin(x(t)), \\ x(t_0) &= x_0 \end{aligned}$$

   with $k = 1$, $t = (0, 6\pi)$, and various initial conditions

$$x_0 = \begin{pmatrix} -1.5 \\ 0 \end{pmatrix}, \begin{pmatrix} -3 \\ 0 \end{pmatrix}, \begin{pmatrix} -\pi \\ 1 \end{pmatrix}.$$

5. Consider the harmonic oscillator

$$\begin{aligned} x''(t) + bx &= c \cos(\omega t), \\ x(t_0) &= x_0 \end{aligned}$$

   (a) Attempt to derive an explicit solution. Consider the two cases $b \neq \omega^2$ and $b = \omega^2$ separately.

   (b) Experiment with the following constants, time range, and initial conditions $x_0$:
   - $a = 0$, $b = 9$, $c = 10$
   - $t = [0, 50]$
   - $x_0 = (1, 0)^\top$
   - $\omega = 2.5, 2.9, 3.1, 3, \sqrt{3}$