Numerical Solution of ODEs

Exercise Class

31st October 2023

Adaptive Timestepping

Algorithm 1. *Given* two *one step methods*

$$\psi \text{ — } \textit{Order } p$$
$$\overline{\psi} \text{ — } \textit{Order } p+1.$$

we define adaptive timestepping at each timestep as:

$\tau \leftarrow \max(\tau, \texttt{TOL})$
$\delta \leftarrow \|\overline{\psi}(t+\tau, t, x) - \psi(t+\tau, t, x)\|$
while $\delta > \texttt{TOL}$ ***do***
  $\tau \leftarrow \tau \left(\frac{\texttt{TOL}}{\delta}\right)^{1/p+1}$
  $\delta \leftarrow \|\overline{\psi}(t+\tau, t, x) - \psi(t+\tau, t, x)\|$
end while
accept τ
$x = \overline{\psi}(t+\tau, t, x)$
$t = t + \tau$

We provide three implementations of an algorithm with $p = 1$:

`ode12_1.m` Basic algorithm (using Euler and Heun)

`ode12_2.m` Adds damping to the timestep size update to prevent large changes.

`ode12.m` Adds heuristics for the initial timestep size.

Exercises

1. Solve

$$x'(t) = \begin{pmatrix} 998 & 1998 \\ -999 & -1999 \end{pmatrix} x(t), \qquad t \in [0, 0.1], \tag{1}$$
$$x(0) = \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \tag{2}$$

implemented by `linsystem.m`, using all three version of ode12 (`ode12_1.m`, `ode12_2.m`, and `ode12.m`).

2. Modify `ode12` to use Euler (order $p = 1$ method) and Runge (order $p+1 = 2$ method). Solve the linear system from (1)–(2) using this method.

3. Study `ode23.m`.