1. Adams-Bashfort $(m=3)$

$$a_3 = 1,\ a_2 = -1,\ a_1 = a_0 = 0,\ b_3 \neq 0$$

$$\ell = 1 \qquad a_1 + 2a_2 + 3a_3 = b_0 + b_1 + b_2 + b_3$$

$$\ell = 2 \qquad a_1 + 4a_2 + 9a_3 = 2(b_1 + 2b_2 + 3b_3)$$

$$\ell = 3 \qquad a_1 + 8a_2 + 27a_3 = 3(b_1 + 4b_2 + 9b_3)$$

$$\ell = 4 \qquad a_1 + 16a_2 + 81a_3 = 4(b_1 + 8b_2 + 27b_3)$$

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 4 & 6 \\ 0 & 3 & 12 & 27 \\ 0 & 4 & 32 & 108 \end{pmatrix} \begin{pmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 5 \\ 19 \\ 65 \end{pmatrix}$$

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 1 \\ 0 & 2 & 4 & 6 & 5 \\ 0 & 3 & 12 & 27 & 19 \\ 0 & 4 & 32 & 108 & 65 \end{array}\right) \to \left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 1 \\ 0 & 2 & 4 & 6 & 5 \\ 0 & 0 & 6 & 18 & 11.5 \\ 0 & 0 & 24 & 96 & 55 \end{array}\right)$$

$$\to \left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 1 \\ 0 & 2 & 4 & 6 & 5 \\ 0 & 0 & 6 & 18 & 11.5 \\ 0 & 0 & 0 & 24 & 9 \end{array}\right) \to \left(\begin{array}{cccc|c} 24 & 24 & 24 & 24 & 24 \\ 0 & 24 & 48 & 72 & 60 \\ 0 & 0 & 24 & 72 & 46 \\ 0 & 0 & 0 & 24 & 9 \end{array}\right)$$

$$\to \left(\begin{array}{cccc|c} 24 & 24 & 24 & 0 & 15 \\ 0 & 24 & 48 & 0 & 33 \\ 0 & 0 & 24 & 0 & 19 \\ 0 & 0 & 0 & 24 & 9 \end{array}\right) \to \left(\begin{array}{cccc|c} 24 & 24 & 0 & 0 & -4 \\ 0 & 24 & 0 & 0 & -5 \\ 0 & 0 & 24 & 0 & 19 \\ 0 & 0 & 0 & 24 & 9 \end{array}\right)$$

$$\to \left(\begin{array}{cccc|c} 24 & 0 & 0 & 0 & 1 \\ 0 & 24 & 0 & 0 & -5 \\ 0 & 0 & 24 & 0 & 19 \\ 0 & 0 & 0 & 24 & 9 \end{array}\right)$$

$$\Rightarrow b_3 = \frac{3}{8},\ b_2 = \frac{19}{24},\ b_1 = \frac{-5}{24},\ b_0 = \frac{1}{24}$$

$$\Rightarrow u_{j+3} = u_{j+2} + \tau\left(\frac{3}{8} f(t_{j+3}, u_{j+3}) + \frac{19}{24} f(t_{j+2}, u_{j+2}) - \frac{5}{24} f(t_{j+1}, u_{j+1}) + \frac{1}{24} f(t_j, u_j)\right).$$

Adams-Moulton $(m=3)$

$$a_3 = 1, \; a_2 = -1, \; a_1 = a_0 = 0, \; b_3 = 0$$

$$\begin{aligned}
l=1 \quad & a_1 + 2a_2 + 3a_3 = b_0 + b_1 + b_2 \\
l=2 \quad & a_1 + 4a_2 + 9a_3 = 2(b_1 + 2b_2) \\
l=3 \quad & a_1 + 8a_2 + 27a_3 = 3(b_1 + 4b_2)
\end{aligned}$$

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 4 \\ 0 & 3 & 12 \end{pmatrix} \begin{pmatrix} b_0 \\ b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 5 \\ 19 \end{pmatrix}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 2 & 4 & 5 \\ 0 & 3 & 12 & 19 \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 2 & 4 & 5 \\ 0 & 0 & 6 & 11.5 \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} 12 & 12 & 12 & 12 \\ 0 & 12 & 24 & 30 \\ 0 & 0 & 12 & 23 \end{array}\right)$$

$$\rightarrow \left(\begin{array}{ccc|c} 12 & 12 & 0 & -11 \\ 0 & 12 & 0 & -16 \\ 0 & 0 & 12 & 23 \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} 12 & 0 & 0 & 5 \\ 0 & 12 & 0 & -16 \\ 0 & 0 & 12 & 23 \end{array}\right)$$

$$\Rightarrow b_2 = \frac{23}{12}, \; b_1 = -\frac{4}{3}, \; b_0 = \frac{5}{12}$$

$$\Rightarrow u_{j+3} = u_{j+2} + \tau\left(\frac{23}{12} f(t_{j+2}, u_{j+2}) - \frac{4}{3} f(t_{j+1}, u_{j+1}) + \frac{5}{12} f(t_j, u_j)\right)$$