Homework 2 — Multi-step Predictor/Corrector Method

Numerical Solution for ODEs

Due date: January 5nd, 2024

Support Files

Support files for this homework can be found as a ZIP file on:

`https://www.karlin.mff.cuni.cz/~congreve/teaching.php?c=WS2023_ODE`

Exercises

Exercise 1. Write a MATLAB function, with the name `pred_corr`, to implement one of the following *predictor/corrector* methods (see support files for initial template):

Algorithm	*Predictor*	*Corrector*
1. $PECE$	3-step Nyström	2-step Milne-Simpson
2. $P(EC)^4$	1-step Adams-Bashfort	3-step Adams-Moulton
3. $P(EC)^3E$	1-step Adams-Bashfort	3-step Adams-Moulton
4. $P(EC)^2E$	2-step Nyström	2-step Milne-Simpson

Exercise 2. Test your script on the following problems from the support files:

1. The logistic equation $x' = (1 - x)x$ (`logistic.m`) for $t \in [0, 3]$, $x_0 = 2$, $\tau = 0.1$ and plot t versus the solution x:

```
x0=2.0; h=0.1;
figure;
[t,x]=pred_corr(@logistic, 0, 3, x0, h);
plot(t,x,'-bx');
```

2. The linear oscillator (`oscillator.m`)

$$x_1' = x_2$$
$$x_2' = -9x_1 + 10\cos(2.5t)$$

for $t \in [0, 10]$, $\boldsymbol{x}_0 = (2, 1)^\top$, $\tau = 0.1$ and plot t versus the solution x_1:

```
figure;
x0 = [2;1]; h = 0.1;
[t,x]=pred_corr(@oscillator, 0, 10, x0, h);
plot(t,x(:,1),'-bx');
```

3. The satellite problem (`sat_ode.m`) with $\mu = \frac{1}{82.45}$

$$
\begin{aligned}
x_1' &= x_3 \\
x_2' &= x_4 \\
x_3' &= 2x_4 + x_1 - (1-\mu)\frac{x_1+\mu}{((x_1+\mu)^2+x_2^2)^{1.5}} - \mu\frac{x_1-1+\mu}{((x_1-1+\mu)^2+x_2^2)^{1.5}} \\
x_4' &= -2x_3 + x_2 - (1-\mu)\frac{x_2}{((x_1+\mu)^2+x_2^2)^{1.5}} - \mu\frac{x_2}{((x_1-1+\mu)^2+x_2^2)^{1.5}}
\end{aligned}
$$

for $t \in [0, 6.192169331319639706 74]$, $\boldsymbol{x}_0 = (1.2, 0, 0, -1.049357509830319907 26)^\top$, $\tau = 0.001$ and plot x_1 versus x_2:

```
figure
x0 = [1.2; 0; 0; -1.04935750983031990726]; h = 1e-3;
[t,x] = pred_corr(@sat_ode, 0, 6.19216933131963970674, x0, h);
plot(x(:,1), x(:,2));
```

Save each of these plots as a PDF file using `Save > Save As`.

Exercise 3. Apply linear regression to estimate the method order. See `conv_analysis.m` for a script to perform this, when called with the `pred_corr`:

```
conv_analysis(@pred_corr);
```

Submission

Submit the MATLAB script for the implemented method from *exercise 1*, the PDF files of the plots from *exercise 2*, and enter the order of the method deduced in *exercise 3* via the *Study Group Roster* (*Záznamník učitele*) in SIS before the deadline.