Numerical Solution of ODEs

Exercise Class

3rd October 2025

Exercises

1. Read about the following ODE systems:

   Population Growth http://en.wikipedia.org/wiki/Population_growth
   Logistic http://en.wikipedia.org/wiki/Logistic_function
   Pendulum http://en.wikipedia.org/wiki/Pendulum_%28mathematics%29
   Harmonic Oscillator http://en.wikipedia.org/wiki/Harmonic_oscillator

2. Consider the logistic equation

$$\begin{aligned} x' &= (a - bx)x - c, \\ x(t_0) &= x_0 \end{aligned}$$

   with non-negative constants a, b, and c. Solve using `ode23` with the right hand side function `logistic.m`, and plot the results, with the following different values:

   constants: $a = b = 1$ and $c = 0$
   time range: $t = [0, 3], [0, -1]$
   initial condition: $x_0 = 1/2, 3/2, 1, -1/20$

3. Compare `ode23` and `ode15s` for the logistic equation with $a = b = 1$, c=$1/5$, $t = [0, 100]$, $x_0 = 0.7233$. Compare also to the known exact solution:

$$u(t) = \frac{\sqrt{5}}{10} \tanh\left((t - t_0)\frac{\sqrt{5}}{10} + \operatorname{arctanh}\left((2x_0 - 1)\sqrt{5} \right) \right) + \frac{1}{2}.$$

4. Use `ode23` to solve the pendulum problem (`pendulum.m`)

$$\begin{aligned} x'' &= -k \sin(x) \qquad \equiv \qquad \begin{cases} x_1' = x_2, \\ x_2' = -k \sin(x) \end{cases} \\ x(t_0) &= x_0 \end{aligned}$$

   with $k = 1$, $t = (0, 6\pi)$, and various initial conditions

$$x_0 = \begin{pmatrix} -1.5 \\ 0 \end{pmatrix}, \begin{pmatrix} -3 \\ 0 \end{pmatrix}, \begin{pmatrix} -\pi \\ 1 \end{pmatrix}.$$

5. Consider the harmonic oscillator

$$\begin{aligned} x'' + bx &= c \cos(\omega t), \\ x(t_0) &= x_0 \end{aligned}$$

(a) Transform into a system of first-order ODEs

(b) Attempt to derive an explicit solution. Consider the two cases $b \neq \omega^2$ and $b = \omega^2$ separately.

(c) Use `ode23` (`oscillator.m`) to solve with:

constants: $a = 0$, $b = 9$, $c = 10$, $\omega = 2.5, 2.9, 3.1, 3, \sqrt{3}$

time range: $t = [0, 50]$

initial condition: $x_0 = (1, 0)^\top$