WP3 – Eigenvalue problems

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Eigenvalue problem for elliptic operators

Laplace eigenvalue problem

$$
\begin{aligned}
-\Delta u_i &= \lambda_i u_i \quad \text{in } \Omega \\
u_i &= 0 \qquad\;\, \text{on } \partial\Omega
\end{aligned}
$$

Two-sided bounds on eigenvalues:

$$\underline{\lambda}_i \leq \lambda_i \leq \overline{\lambda}_i$$

[Goerisch, Haunhorst 1985], [Kato 1949], [Lehmann 1949, 1950], [Barrenechea, Boulton, Boussaïd 2014], [Cancès, Dusson, Maday, Stamm, Vohralík 2017, 2018, 2019], [Carstensen, Gedicke 2014], [Carstensen, Gallistl 2014], [Hu, Huang, Lin 2014], [Liu 2015], [Liu, Oishi 2013], [Šebestová, V. 2014], [V. 2018], and many others.

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Approximate eigenfunctions: $\hat{u}_i$

Goal: fully computable error bound for eigenfuntions

$$\|\nabla u_i - \nabla \hat{u}_i\| \le \eta(\underline{\lambda}_i, \overline{\lambda}_i, \hat{u}_i)$$

Error bounds on eigenfunctions

Problem: Eigenfunctions may be ill-posed.

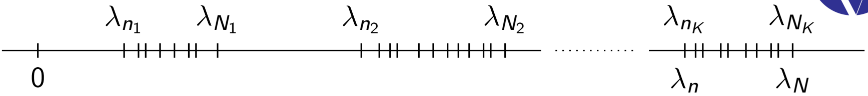
Solution:
(a) consider spaces of eigenfunctions

- $\lambda_n, \lambda_{n+1}, \ldots, \lambda_N$ (cluster)
- $E = \text{span}\{u_n, u_{n+1}, \ldots, u_N\}$ (space of eigenfunctions)
- $\widehat{E} = \text{span}\{\hat{u}_n, \hat{u}_{n+1}, \ldots, \hat{u}_N\}$ (its approximation)

(b) bound the directed distance of spaces

- $\delta(E, \widehat{E}) \leq \eta(\underline{\lambda}_i, \overline{\lambda}_i, \hat{u}_i)$ [Meyer 2000]

The bound on eigenfunctions in energy norm



Directed distance: $\delta(E_K, \widehat{E}_K) = \max\limits_{\substack{v \in E_K \\ \|\nabla v\|=1}} \min\limits_{\hat{v} \in \widehat{E}_K} \|\nabla v - \nabla \hat{v}\|$

Nonorthogonality: $\hat{\zeta}(\widehat{E}_k, \widehat{E}_K) = \max\limits_{\substack{v \in \widehat{E}_k \\ \|\nabla v\|=1}} \max\limits_{\substack{w \in \widehat{E}_K \\ \|\nabla w\|=1}} (\nabla v, \nabla w)$

Theorem 1 (X. Liu, T.V.)

$$\delta^2(E_K, \widehat{E}_K) \leq \frac{\rho\left(\hat{\lambda}_N^{(K)} - \underline{\lambda}_n\right) + \overline{\lambda}_n \hat{\lambda}_N^{(K)} \vartheta^{(K)}}{\hat{\lambda}_N^{(K)}(\rho - \overline{\lambda}_n)},$$

where $\overline{\lambda}_n < \rho \leq \underline{\lambda}_{N+1}$,

$$\hat{\lambda}_N^{(K)} = \max_{\hat{v} \in \widehat{E}_K} \frac{\|\nabla \hat{v}\|^2}{\|\hat{v}\|^2}, \quad \vartheta^{(K)} = \sum_{k=1}^{K-1} \frac{\rho - \lambda_{n_k}}{\lambda_{n_k}} \left[\hat{\zeta}(\widehat{E}_k, \widehat{E}_K) + \delta(E_k, \widehat{E}_k)\right]^2.$$

WP3 – The plan

Task 3.1. Guaranteed error bounds for eigenfunctions (TV, PT, JP)

- 1st paper: ready for submission (with X. Liu)
- 2nd paper: an idea exists (proof technique of the LG method)
- 3rd paper: LG method with equilibrated flux reconstruction (with JP)

Task 3.2. Lower bounds on eigenvalues by DGM (TV, VD, JP)

- Risky, interesting. It needs to be tested first.
- Planned to 2021, but we need to start already this year.
- Collaboratoin with a DG expert needed.

Task 3.3. Adaptivity for eigenvalue problems (TV, PK, JŠ, MK)

- Parallel mesh adaptivity (T4.5) for eigenvalue problems
- Mesh refinement driven by flux reconstructions used in T3.1 or by any other error estimator.
- Possible problem: tight clusters (or multiple eigenvalues)
- Planned to 2022.

WP1 – Time independent problems

Task 1.4. Error estimation for convection-diffusion problems

(PK, TV, BS)

- Meeting on March 3, 2020.

Thank you for your attention

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